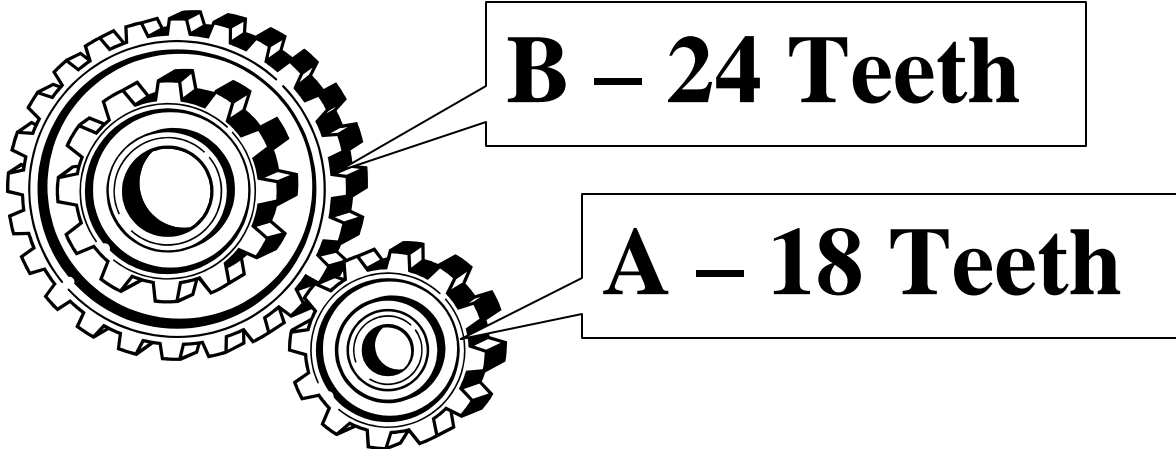


GEARING UP EXAMPLES



DEFINITION - RATIO

As gear A revolves 4 times, it will cause gear B to revolve 3 times. Hence, we say that gear ratio of A to B is 4 to 3. In mathematics, a ratio is a comparison of two numbers by division. The gear ratio above can be expressed in the following ways:

$$4 \text{ to } 3 \quad 4:3 \quad \frac{4}{3}$$

A ratio is commonly expressed as a fraction in simplest form!

Example 1: What is the ratio of gear A to gear B if gear A has 10 teeth to 6 teeth (in fraction form)?

The ratio is $\frac{10}{6}$ or $\frac{5}{3}$.

Example 2: What is the ratio of 20 inches to 4 feet?

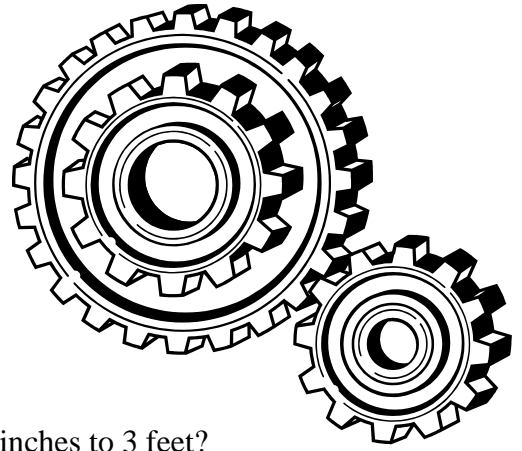
The units should be the same!

$$4 \text{ feet} = 48 \text{ inches}$$

The ratio is $\frac{20in}{48in}$ or $\frac{5}{12}$.

Example 3: What is the gear ratio, as a fraction, for 150 teeth to 25 teeth?

The ratio is $\frac{150}{25} = \frac{6}{1} = 6$.



Example 4: What is the ratio, as a fraction, for 16 inches to 3 feet?

The units must be the same!

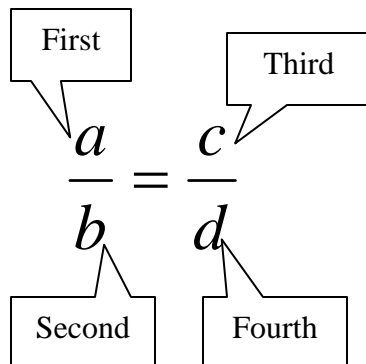
3 feet = 36 inches

The ratio is $\frac{16in}{36in} = \frac{4}{9}$.

DEFINITION – PROPORTION

An equation of the form $\frac{a}{b} = \frac{c}{d}$ that states that two ratios are equal is called a proportion.

Every proportion consists of four terms.



The first and fourth terms, a and d, are called the extremes.

The second and third terms, b and c, are called the means.

MEANS-EXTREMES PROPERTY OF PROPORTIONS:

In a proportion, the product of the extremes is equal to the product of the means.

If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

You can use this property to solve some equations. The equations must be in the form of a proportion.

Example 5: Solve: $\frac{21}{27} = \frac{x}{18}$

$$\frac{21}{27} = \frac{x}{18}$$
$$21(18) = 27x$$
$$378 = 27x$$
$$14 = x$$

**Means-Extremes
Property**

Example 6: Solve: $\frac{x}{5} = \frac{x+3}{10}$

$$\frac{x}{5} = \frac{x+3}{10}$$
$$x(10) = 5(x+3)$$
$$10x = 5x + 5(3)$$
$$10x = 5x + 15$$
$$10x - 5x = 5x - 5x + 15$$
$$5x = 15$$
$$5x \div 5 = 15 \div 5$$
$$x = 3$$

**Means-Extremes
Property**

Example 7: Write a proportion for the following word problem, then use the 4-step approach to problem solving to solve the problem.

A gear of 96 teeth required a matching gear of 6 teeth. At that rate, how many teeth would be needed on gear B if gear A had 152 teeth? (Round answer to nearest whole number of teeth.)

Explore:

Let x = teeth required for gear B

Plan:

Write a proportion for the problem.

$$\frac{96}{6} = \frac{152}{x}$$

Solve:

$$\frac{96}{6} = \frac{152}{x}$$

$$96x = 912$$

$$96x \div 96 = 912 \div 96$$

$$x = 9 \frac{1}{2} \text{ or } 9.5$$

A gear of 10 teeth is required.

Examine:

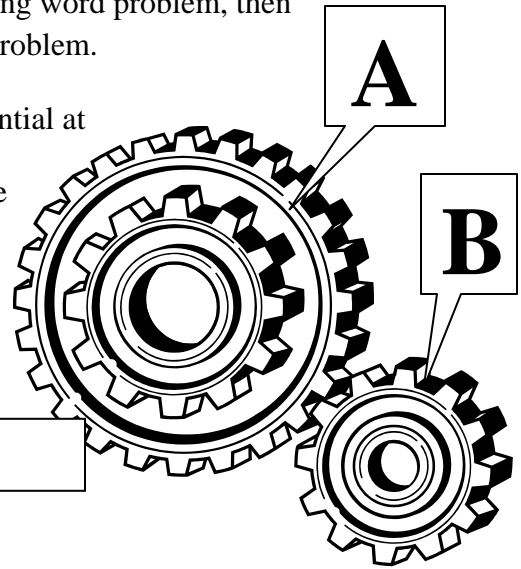
Gear B is $1/16$ as large as gear A.

Is $(1/16)(152) = 10$ (approximately)?

(YES)

Example 8: Write a proportion for the following word problem, then use the 4-step approach to problem solving to solve the problem.

In the diagram at the right, gear A connects to the differential at gear B. There are 21 teeth on gear A and 14 teeth on Gear B. It has been decided that the gear ratio needs to be increased to a 5 to 2 (A:B) ratio. If gear B remains the same, how many teeth must gear A have?



Explore:

Let x = teeth on gear A after change

Plan:

Write a proportion for the problem.

$$\frac{x}{14} = \frac{5}{2}$$

Solve:

$$\frac{x}{14} = \frac{5}{2}$$

$$2x = 70$$

$$2x \div 2 = 70 \div 2$$

$$x = 35$$

A new gear would require 35 teeth.

Examine:

Gear B must retain 14 teeth.
Gear A must be in the ratio of 5:2 times larger.
Is $14(5/2) = 35$? (YES)

Name: _____

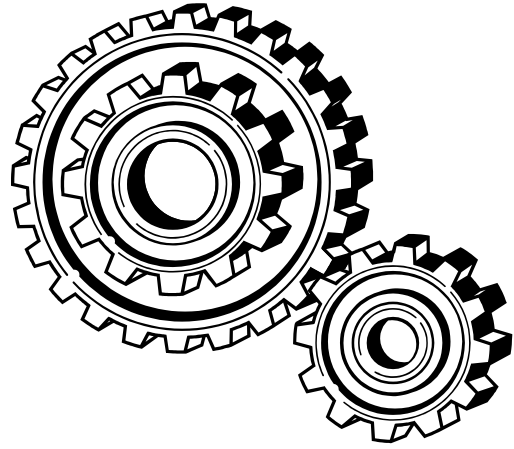
Date: _____

Class: _____

GEARING UP WORKSHEET

Write each ratio as a fraction in simplest form:

1. 3 gears to 11 gears
2. 7 gears to 3 gears
3. 21 feet to 16 feet
4. 16 cm to 5 cm
5. 21 ounces to 6 ounces
6. 15 km to 5 km
7. 8 feet to 28 inches
8. 4 pounds to 100 ounces
9. 16 cm to 40 mm
10. 72 mm to 90 mm
11. 35 minutes to 2 hours
12. 5 hours to 52 minutes



Show steps to solve each proportion.

$$13. \frac{3}{4} = \frac{x}{8}$$

$$14. \frac{3}{15} = \frac{x}{45}$$

$$15. \frac{2}{10} = \frac{1}{x}$$

$$16. \frac{x+2}{5} = \frac{7}{5}$$

$$17. \frac{6}{14} = \frac{7}{x-3}$$

$$18. \frac{3}{5} = \frac{x+2}{6}$$

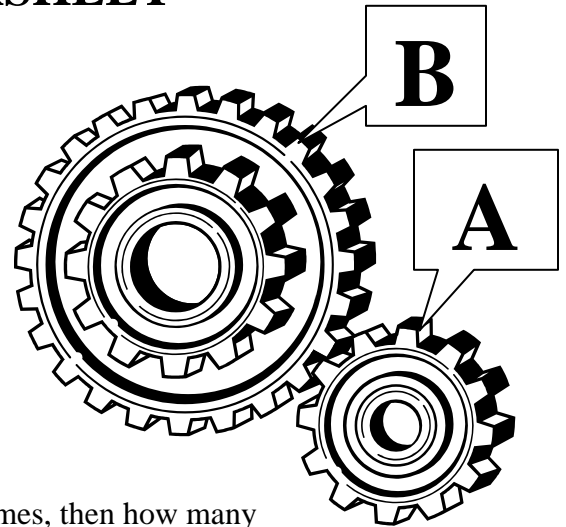
$$19. \frac{14}{10} = \frac{5+x}{x-3}$$

$$20. \frac{9}{x-8} = \frac{4}{5}$$

GEARING UP WORKSHEET

For each problem use the 4-step approach to problem solving:

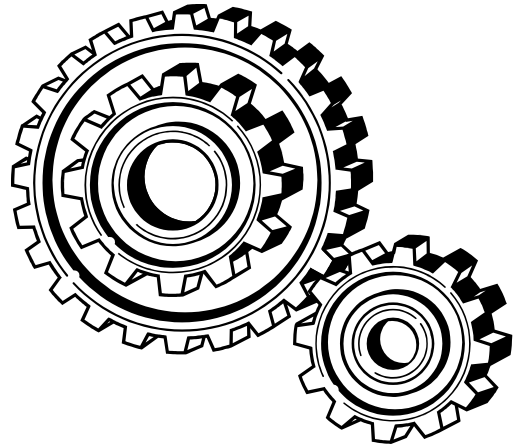
- a. Explore “Define a variable”
- b. Plan “Write an equation”
- c. Solve “Solve the equation and answer the problem”
- d. Examine “Check to see if the answer makes sense”



21. If gear A turns 5 times each time gear B turns 2 times, then how many times will Gear A turn if Gear B turns 10 times?
22. If gear A turns 7 times each time gear B turns $1\frac{1}{2}$ times, then how many times will Gear B turn if Gear A turns 12 times?
23. Stewart earns \$97 in 4 days. At that rate, how many days will it take him to earn \$485?
24. Shawn used 25 gallons of gasoline in traveling 350 miles. How much gasoline will be used in traveling 462 miles?
25. One gear has 36 teeth. The ratio of this gear to a second gear is 4 to 3. How many teeth does the second gear have?

GEARING UP WORKSHEET KEY

Write each ratio as a fraction in simplest form:



- | | | |
|-----------------|---------------------------|-----------------|
| $\frac{7}{3}$ | 1. 3 gears to 11 gears | $\frac{3}{11}$ |
| | 2. 7 gears to 3 gears | |
| | 3. 21 feet to 16 feet | $\frac{21}{16}$ |
| $\frac{16}{5}$ | 4. 16 cm to 5 cm | |
| | 5. 21 ounces to 6 ounces | $\frac{7}{2}$ |
| $\frac{3}{1}$ | 6. 15 km to 5 km | $\frac{24}{7}$ |
| | 7. 8 feet to 28 inches | |
| $\frac{16}{25}$ | 8. 4 pounds to 100 ounces | $\frac{4}{1}$ |
| | 9. 16 cm to 40 mm | |
| $\frac{4}{5}$ | 10. 72 mm to 90 mm | |
| | 11. 35 minutes to 2 hours | $\frac{7}{24}$ |
| $\frac{75}{13}$ | 12. 5 hours to 52 minutes | |

Show steps to solve each proportion:

13. $\frac{3}{4} = \frac{x}{8}$	$\frac{3}{4} = \frac{x}{8}$ $3(8) = 4x$ $24 = 4x$ $6 = x$
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14. $\frac{3}{15} = \frac{x}{45}$	$\frac{3}{15} = \frac{x}{45}$ $3(45) = 15x$ $135 = 15x$ $9 = x$
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GEARING UP WORKSHEET KEY

15. $\frac{2}{10} = \frac{1}{x}$

$$\begin{aligned} \frac{2}{10} &= \frac{1}{x} \\ 2x &= 1(10) \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

16. $\frac{x+2}{5} = \frac{7}{5}$

$$\begin{aligned} \frac{x+2}{5} &= \frac{7}{5} \\ (x+2)(5) &= 7(5) \\ 5x+10 &= 35 \\ 5x &= 25 \\ x &= 5 \end{aligned}$$

17. $\frac{6}{14} = \frac{7}{x-3}$

$$\begin{aligned} \frac{6}{14} &= \frac{7}{x-3} \\ 6(x-3) &= 7(14) \\ 6x-18 &= 98 \\ 6x &= 116 \\ x &= 19 \frac{1}{3} \end{aligned}$$

18. $\frac{3}{5} = \frac{x+2}{6}$

$$\begin{aligned} \frac{3}{5} &= \frac{x+2}{6} \\ 3(6) &= 5(x+2) \\ 18 &= 5x+10 \\ 8 &= 5x \\ 1 \frac{3}{5} &= x \end{aligned}$$

19. $\frac{14}{10} = \frac{5+x}{x-3}$

$$\begin{aligned} \frac{14}{10} &= \frac{5+x}{x-3} \\ 14(x-3) &= 10(5+x) \\ 14x-42 &= 50+10x \\ 4x-42 &= 50 \\ 4x &= 92 \\ x &= 23 \end{aligned}$$

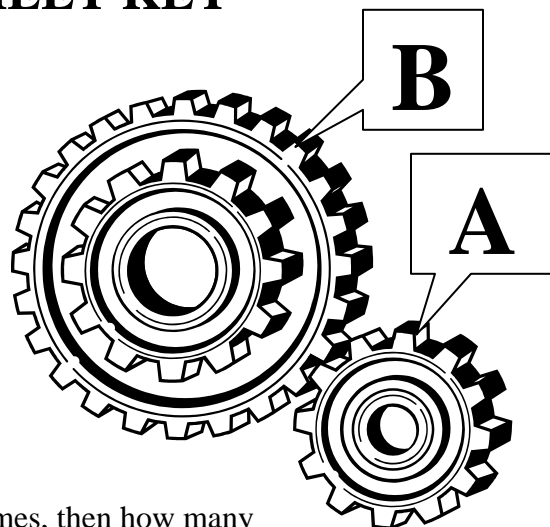
20. $\frac{9}{x-8} = \frac{4}{5}$

$$\begin{aligned} \frac{9}{x-8} &= \frac{4}{5} \\ 9(5) &= 4(x-8) \\ 45 &= 4x-32 \\ 77 &= 4x \\ 19 \frac{1}{4} &= x \end{aligned}$$

GEARING UP WORKSHEET KEY

For each problem use the 4-step approach to problem solving:

- a. Explore “Define a variable”
- b. Plan “Write an equation”
- c. Solve “Solve the equation and answer the problem”
- d. Examine “Check to see if the answer makes sense”



20. If gear A turns 5 times each time gear B turns 2 times, then how many times will Gear A turn if Gear B turns 10 times?

Let x be the number times gear A will turn.

$$\frac{5}{2} = \frac{x}{10}$$

$$5(10) = 2x \rightarrow 50 = 2x \rightarrow 25 = x$$

If gear B turns 10 times then this is 5 times larger. Is 25 five times larger than 5? (YES)

21. If gear A turns 7 times each time gear B turns $1\frac{1}{2}$ times, then how many times will Gear B turn if Gear A turns 12 times?

Let x be the number of times gear B turns.

$$\frac{7}{1\frac{1}{2}} = \frac{12}{x}$$

$$7x = (12)(1\frac{1}{2}) \rightarrow 7x = 18 \rightarrow x = 2\frac{4}{7}$$

If gear A turns 12 times then this is a little less than twice as many turns. Is 2 times ($1\frac{1}{2}$) approximately $2\frac{4}{7}$? (YES)

22. Stewart earns \$97 in 4 days. At that rate, how many days will it take him to earn \$485?

Let x be number of days to earn \$486.

$$\frac{97}{4} = \frac{485}{x}$$

$$97x = 1940 \rightarrow x = 20$$

\$485 is 5 times greater than \$97. Then it should take 5 times longer to earn the needed money. Is $5(4) = 20$? (YES)

GEARING UP WORKSHEET KEY

23. Shawn used 25 gallons of gasoline in traveling 350 miles. How much gasoline will be used in traveling 462 miles?

Let x be the amount of gasoline used.

$$\frac{25}{350} = \frac{x}{462}$$

$$25(462) = 350x \rightarrow 11550 = 350x \rightarrow 33 = x$$

462 miles is a little less than $1 \frac{1}{2}$ times larger than 350 miles. Is 33 approximately $1 \frac{1}{2}$ times larger than 25? (YES)

24. One gear has 36 teeth. The ratio of this gear to a second gear is 4 to 3. How many teeth does the second gear have?

Let x be the number of teeth the second gear has.

$$\frac{4}{3} = \frac{36}{x}$$

$$4x = 3(36) \rightarrow 4x = 108 \rightarrow x = 27$$

Is the ratio of 36 teeth to 27 teeth the same as 4 to 3? (YES)

Student Name: _____

Date: _____

GEARING UP CHECKLIST


1. On each problem, did the student write each ratio as a fraction in simplest form?
 - a. All twelve (30 points)
 - b. Ten or more of the twelve (25 points)
 - c. Eight or more of the twelve (20 points)
 - d. Six or more of the twelve (15 points)
 - e. Four or more of the twelve (10 points)
 - f. Two or more of the twelve (5 points)

2. On each problem, did the student show steps to solve each proportion correctly?
 - a. All eight (30 points)
 - b. Seven of the eight (25 points)
 - c. Six of the eight (20 points)
 - d. Five of the eight (15 points)
 - e. Four of the eight (10 points)
 - f. Three of the eight (5 points)

3. On each problem, did the student use the 4-step approach to problem solving?
 - a. All five (30 points)
 - b. Four of the five (25 points)
 - c. Three of the five (20 points)
 - d. Two of the five (15 points)
 - e. One of the five (10 points)

Total Number of Points _____

- A 81 points and above
- B 72 points and above
- C 63 points and above
- D 54 points and above
- F 53 points and below



**Any score below C
needs
remediation!**