PERFECT SQUARES AND FACTORING EXAMPLES

1. Ask the students what is meant by “identical.” Get their responses and then explain that when we have two factors that are “identical,” we call them “perfect squares.” The term “perfect” and “identical” are synonyms for each other in this case.

2. Ask students to find the product for each of the following. (Encourage students to look for patterns.)

   a) \((x + 3)^2\)
   \[
   (x + 3)(x + 3) = x^2 + 2(3)(x) + (3)^2 = x^2 + 6x + 9
   \]

   b) \((x - 3)^2\)
   \[
   (x - 3)(x - 3) = x^2 + 2(-3)(x) + (-3)^2 = x^2 - 6x + 9
   \]

   c) \((2x + 4)^2\)
   \[
   (2x + 4)(2x + 4) = 4x^2 + 16x + 16
   \]

3. Help students recognize the following “Perfect Square” patterns from the examples above!

   \[
   (a + b)^2 = a^2 + 2ab + b^2
   \]
   \[
   (a - b)^2 = a^2 - 2ab + b^2
   \]
4. These patterns can help you factor trinomials, such as $y^2 + 16y + 64$ and $4x^2 – 20xy + 25y^2$. (Help students see similarities.)

Finding a Product

$$(y + 8)^2 = y^2 + 2(y)(8) + (8)^2$$

$\Rightarrow y^2 + 16y + 64$

Factoring

$$y^2 + 16y + 64 = (y)^2 + 2(y)(8) + (8)^2$$

$$\Rightarrow (y + 8)^2$$

Finding a Product

$$(2x – 5y)^2 = (2x)^2 - 2(2x)(5y) + (5y)^2$$

$\Rightarrow 4x^2 – 20xy + 25y^2$

Factoring

$$4x^2 – 20xy + 25y^2 = (2x)^2 - 2(2x)(5y) + (5y)^2$$

$$\Rightarrow (2x – 5y)^2$$

5. Caution students against confusing the “difference of squares” with the “square of a difference.”

$$(a + b)(a – b) = a^2 – b^2$$

$$(a – b)^2 \neq a^2 – b^2$$

6. To determine whether a trinomial can be factored in this way, first decide if it is a perfect square. In other words, decide if it can be written in either of these forms:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a – b)^2 = a^2 – 2ab + b^2$$
7. Example: Determine whether \(x^2 + 22x + 121\) is a perfect square. If it is, factor it.

To determine whether \(x^2 + 22x + 121\) is a perfect square, answer each question.

a. Is \(x^2\) a perfect square \(\Rightarrow x^2 = (x)^2\) (YES)

b. Is 121 a perfect square \(\Rightarrow 121 = (11)^2\) (YES)

c. Is the middle term twice the product of (x) and (11) \(\Rightarrow 22x = 2(x)(11)\) (YES)

Since all three answers are “YES,” the trinomial \(x^2 + 22x + 121\) is a perfect square. It can be factored as follows:

\[
x^2 + 22x + 121 \Rightarrow (x)^2 + 2(11)(x) + (11)^2
\]

\[
(x + 11)^2
\]

---

8. Steps a, b, and c give students a system for determining perfect square trinomials.

9. Point out that before factoring a polynomial, its terms should be arranged so that the powers of “x” are in descending or ascending order. For example, \(8x + x^2 + 16\) should be written as \(x^2 + 8x + 16\) or \(16 + 8x + x^2\).

10. Example: Determine whether \(16a^2 + 72a + 81\) is a perfect square. If it is, factor it.

To determine whether \(16a^2 + 72a + 81\) is a perfect square, answer each question.

d. Is \(16a^2\) a perfect square \(\Rightarrow 16a^2 = (4a)^2\) (YES)

e. Is 81 a perfect square \(\Rightarrow 81 = (9)^2\) (YES)

f. Is the middle term twice the product of \(4a\) and \(9\) \(\Rightarrow 72a = 2(4a)(9)\) (YES)

Since all three answers are “YES,” the trinomial \(16a^2 + 72a + 81\) is a perfect square. It can be factored as follows:

\[
16a^2 + 72a + 81 \Rightarrow (4a)^2 + 2(4a)(9) + (9)^2
\]

\[
(4a + 9)^2
\]
11. Example: Determine whether $15 + 4a^2 - 20a$ is a perfect square. If it is, factor it.

To determine whether $15 + 4a^2 - 20a$ is a perfect square, first arrange the terms so that the powers of “a” are in descending order.

$15 + 4a^2 - 20a \rightarrow 4a^2 - 20a + 15$

g. Is $4a^2$ a perfect square $\Rightarrow 4a^2 = (2a)^2$ (YES)
h. Is $15$ a perfect square $\Rightarrow 15 = (?)^2$ (NO)

Since step (b) is (NO), then we stop and rule out the “perfect square” model.

12. Example: Determine whether $16x^2 - 26x + 49$ is a perfect square. If it is, factor it.

To determine whether $16x^2 - 26x + 49$ is a perfect square, answer each question.

i. Is $16x^2$ a perfect square $\Rightarrow 16x^2 = (4x)^2$ (YES)
j. Is $49$ a perfect square $\Rightarrow 49 = (7)^2$ (YES)
k. Is the middle term twice the product of $(4x)$ and $(7)$ $\Rightarrow 26x = 2(4x)(7)$ (NO)

Since “c” is (NO) then $16x^2 - 26x + 49$ is not a perfect square.

13. Example: Determine whether $9x^2 - 12xy + 4y^2$ is a perfect square. If it is, factor it.

To determine whether $9x^2 - 12xy + 4y^2$ is a perfect square. Answer each question.

l. Is $9x^2$ a perfect square $\Rightarrow 9x^2 = (3x)^2$ (YES)
m. Is $4y^2$ a perfect square $\Rightarrow 4y^2 = (2y)^2$ (YES)
n. Is the middle term twice the product of $(3x)$ and $(2y)$ $\Rightarrow 12xy = 2(3x)(2y)$ (YES)

Since all three answers are “YES,” the trinomial $9x^2 - 12xy + 4y^2$ is a perfect square. It can be factored as follows:

$9x^2 - 12xy + 4y^2 \Rightarrow (3x)^2 - 2(3x)(2y) + (2y)^2$
$\Rightarrow (3x - 2y)^2$
PERFECT SQUARES AND FACTORING WORKSHEET

Determine whether each trinomial is a perfect square trinomial. If it is, factor it. (Demonstrate using the 3-step model.)

1. \(a^2 + 4a + 4\)

2. \(x^2 - 10x - 100\)

3. \(n^2 - 13n + 36\)

4. \(y^2 - 8y + 10\)

5. \(4x^2 - 4x + 1\)

6. \(9b^2 - 6b + 1\)

7. \(a^2 + 12a + 36\)

8. \(n^2 - 8n + 16\)

9. \(x^2 + 6x - 9\)

10. \(121y^2 + 22y + 1\)
PERFECT SQUARES AND FACTORING WORKSHEET

Determine whether each trinomial is a perfect square trinomial. If it is, factor it.
(Demonstrate using 3-step model.)

1. $a^2 + 4a + 4$

<table>
<thead>
<tr>
<th>To determine whether $a^2 + 4a + 4$ is a perfect square, answer each question.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Is $a^2$ a perfect square $\Rightarrow a^2 = (a)^2$ (YES)</td>
</tr>
<tr>
<td>b. Is 4 a perfect square $\Rightarrow 4 = (2)^2$ (YES)</td>
</tr>
<tr>
<td>c. Is the middle term twice the product of (a) and (2) $\Rightarrow 4a = 2(a)(2)$ (YES)</td>
</tr>
</tbody>
</table>

Since all three answers are “YES,” the trinomial $a^2 + 4a + 4$ is a perfect square. It can be factored as follows:

$$a^2 + 4a + 4 \Rightarrow (a)^2 + 2(a)(2) + (2)^2 \Rightarrow (a + 2)^2$$

2. $x^2 – 10x – 100$

<table>
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<tr>
<th>To determine whether $x^2 – 10x – 100$ is a perfect square, answer each question.</th>
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<tbody>
<tr>
<td>d. Is $x^2$ a perfect square $\Rightarrow x^2 = (x)^2$ (YES)</td>
</tr>
<tr>
<td>e. Is –100 a perfect square $\Rightarrow –100 = (?)^2$ (NO)</td>
</tr>
</tbody>
</table>

Since (b) is (NO) the trinomial $x^2 – 10x – 100$ is not a perfect square.

3. $n^2 – 13n + 36$

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</thead>
<tbody>
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<td>f. Is $n^2$ a perfect square $\Rightarrow n^2 = (n)^2$ (YES)</td>
</tr>
<tr>
<td>g. Is 36 a perfect square $\Rightarrow 36 = (6)^2$ (YES)</td>
</tr>
<tr>
<td>h. Is the middle term twice the product of (n) and (6) $\Rightarrow 13a = 2(n)(6)$ (NO)</td>
</tr>
</tbody>
</table>

Since (c) is (NO) the trinomial $n^2 – 13n + 36$ is not a perfect square.
4. \(y^2 - 8y + 10\)

To determine whether \(y^2 - 8y + 10\) is a perfect square, answer each question.

i. Is \(y^2\) a perfect square \(\Rightarrow y^2 = (y)^2\) (YES)

j. Is 10 a perfect square \(\Rightarrow 10 = (?)^2\) (NO)

Since (b) is (NO), the trinomial \(y^2 - 8y + 10\) is not a perfect square. It can be factored as follows:

\[y^2 - 8y + 10 = (y - 4)^2\]

5. \(4x^2 - 4x + 1\)

To determine whether \(4x^2 - 4x + 1\) is a perfect square, answer each question.

k. Is \(4x^2\) a perfect square \(\Rightarrow 4x^2 = (2x)^2\) (YES)

l. Is 1 a perfect square \(\Rightarrow 1 = (1)^2\) (YES)

m. Is the middle term twice the product of \(2x\) and \(1\) \(\Rightarrow 4x = 2(2x)(1)\) (YES)

Since all three answers are “YES,” the trinomial \(4x^2 - 4x + 1\) is a perfect square. It can be factored as follows:

\[4x^2 - 4x + 1 = (2x - 1)^2\]

6. \(9b^2 - 6b + 1\)

To determine whether \(9b^2 - 6b + 1\) is a perfect square, answer each question.

n. Is \(9b^2\) a perfect square \(\Rightarrow 9b^2 = (3b)^2\) (YES)

o. Is 1 a perfect square \(\Rightarrow 1 = (1)^2\) (YES)

p. Is the middle term twice the product of \(2x\) and \(1\) \(\Rightarrow 6b = 2(3b)(1)\) (YES)

Since all three answers are “YES,” the trinomial \(9b^2 - 6b + 1\) is a perfect square. It can be factored as follows:

\[9b^2 - 6b + 1 = (3b - 1)^2\]
7. \( a^2 + 12a + 36 \)

To determine whether \( a^2 + 12a + 36 \) is a perfect square, answer each question.

q. Is \( a^2 \) a perfect square \( \Rightarrow a^2 = (a)^2 \) (YES)

r. Is 36 a perfect square \( \Rightarrow 36 = (6)^2 \) (YES)

s. Is the middle term twice the product of \( a \) and \( 6 \) \( \Rightarrow 12a = 2(a)(6) \) (YES)

Since all three answers are “YES,” the trinomial \( a^2 + 12a + 36 \) is a perfect square. It can be factored as follows:

\[
(a + 6)^2
\]

8. \( n^2 – 8n + 16 \)

To determine whether \( n^2 – 8n + 16 \) is a perfect square, answer each question.

t. Is \( n^2 \) a perfect square \( \Rightarrow n^2 = (n)^2 \) (YES)

u. Is 16 a perfect square \( \Rightarrow 16 = (4)^2 \) (YES)

v. Is the middle term twice the product of \( n \) and \( 4 \) \( \Rightarrow 8n = 2(n)(4) \) (YES)

Since all three answers are “YES,” the trinomial \( n^2 – 8n + 16 \) is a perfect square. It can be factored as follows:

\[
(a – 4)^2
\]

9. \( x^2 + 6x – 9 \)

To determine whether \( x^2 + 6x – 9 \) is a perfect square, answer each question.

w. Is \( x^2 \) a perfect square \( \Rightarrow x^2 = (x)^2 \) (YES)

x. Is –9 a perfect square \( \Rightarrow -9 = (?)^2 \) (NO)

Since (c) is (NO), the trinomial \( x^2 + 6x – 9 \) is not a perfect square.
10. $121y^2 + 22y + 1$

To determine whether $121y^2 + 22y + 1$ is a perfect square, answer each question.

y. Is $121y^2$ a perfect square $\Rightarrow 121y^2 = (11y)^2$ (YES)
z. Is 1 a perfect square $\Rightarrow 1 = (1)^2$ (YES)

aa. Is the middle term twice the product of $(11y)$ and 1

$\Rightarrow 22y = 2(11y)(1)$ (YES)

Since all three answers are “YES,” the trinomial $121y^2 + 22y + 1$ is a perfect square. It can be factored as follows:

$$121y^2 + 22y + 1 \Rightarrow (11y)^2 + 2(11y)(1) + (1)^2$$

$$\Rightarrow (11y + 1)^2$$
PERFECT SQUARES AND FACTORING CHECKLIST

1. On questions 1 thru 10, did the student determine if the trinomial is a perfect square trinomial?
   a. All ten (40 points)
   b. Nine of the ten (35 points)
   c. Eight of the ten (30 points)
   d. Seven of the ten (25 points)
   e. Six of the ten (20 points)
   f. Five of the ten (15 points)
   g. Four of the ten (10 points)
   h. Three of the ten (5 points)

2. On question 1, did the student factor the trinomial completely using the 3-step model?
   a. Yes (10 points)
   b. Factored but not completely or did not use 3-step model

3. On question 2, did the student factor the trinomial completely using the 3-step model?
   a. Yes (10 points)
   b. Factored but not completely or did not use 3-step model

4. On question 3, did the student factor the trinomial completely using the 3-step model?
   a. Yes (10 points)
   b. Factored but not completely or did not use 3-step model

5. On question 4, did the student factor the trinomial completely using the 3-step model?
   a. Yes (10 points)
   b. Factored but not completely or did not use 3-step model

6. On question 5, did the student factor the trinomial completely using the 3-step model?
   a. Yes (10 points)
   b. Factored but not completely or did not use 3-step model
7. On question 6, did the student factor the trinomial completely using the 3-step model?
   a. Yes (10 points)
   b. Factored but not completely or did not use 3-step model

8. On question 7, did the student factor the trinomial completely using the 3-step model?
   a. Yes (10 points)
   b. Factored but not completely or did not use 3-step model

9. On question 8, did the student factor the trinomial completely using the 3-step model?
   a. Yes (10 points)
   b. Factored but not completely or did not use 3-step model

10. On question 9, did the student factor the trinomial completely using the 3-step model?
    a. Yes (10 points)
    b. Factored but not completely or did not use 3-step model

11. On question 10, did the student factor the trinomial completely using the 3-step model?
    a. Yes (10 points)
    b. Factored but not completely or did not use 3-step model

Total Number of Points __________

A  126 points and above
B  112 points and above
C  98 points and above
D  84 points and above
F  84 points and below

Any score below C needs remediation!