

MAXIMUM PROFIT EXAMPLES

1. Many times business will raise the prices of their goods or services to increase their profit. However, when they raise their prices, they usually lose some customers. In such situations, the price at which the “maximum” profit occurs needs to be found.
2. **Example:** An auditorium has seats for 1200 people. For the past several days, the auditorium has been filled to capacity for each show. Tickets currently cost \$5.00 and the owner wants to increase the ticket prices. He estimates that for each \$0.50 increase in price, 100 fewer people will attend. What ticket price will maximize the profit?

**Let x = number of \$0.50 price increases.
Thus $5.00 + 0.50x$ represents the single price and
 $1200 - 100x$ represents the number of tickets sold.**

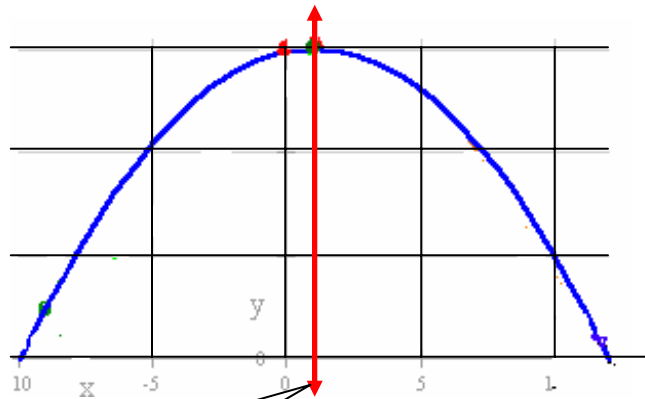
**Income = (number of tickets sold)(ticket price)
Income = $(1200 - 100x)(5.00 + 0.50x)$
Income = $6000 + 100x - 50x^2$**

**Notice that the result is a quadratic equation.
The graph of the related function, $y = -50x^2 + 100x + 6000$,
opens downward and thus has a maximum point. Since
this is a maximum point, the x-coordinate gives the
number of price increases needed to maximize the profit.**

**Recall that the x-coordinate of the maximum point
is given by the equation of the axis of symmetry.**

$$x = -\frac{b}{2a}$$
$$x = -\frac{100}{2(-50)}$$
$$x = 1$$

**The equation of the axis
of symmetry is $x = 1$.**



Axis of symmetry
 $x = 1$

Thus, the profit is maximized when the owner makes one \$0.50 increase.

$$\text{Single ticket price} = 5.00 + 0.50x$$

$$\text{Single ticket price} = 5.00 + 0.50(1)$$

$$\text{Single ticket price} = \$5.50$$

$$\text{Maximum income} = (\text{number of tickets sold})(5.00 + 0.50x)$$

$$\text{Maximum income} = (1200 - 100x)(5.00 + 0.50x)$$

$$\text{Maximum income} = (1200 - 100)(5.00 + 0.50)$$

$$\text{Maximum income} = (1100)(5.50)$$

$$\text{Maximum income} = \$6050$$

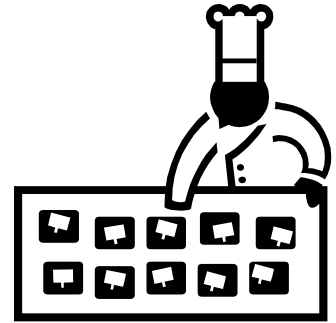
Name: _____

Date: _____

Class: _____

MAXIMUM PROFIT WORKSHEET

1. A grocer sells 50 loaves of bread a day. The cost is \$0.65 a loaf. The grocer estimates that for each \$0.05 price increase, 2 fewer loaves of bread will be sold. Graph, and then determine what cost will maximize the profit?



2. A bus company transports 500 people a day between Morse Rd. and high St. The one-way fare is \$0.50. The owners estimates that for each \$0.10 price increase, 50 passengers will be lost. Graph, then determine what price will maximize their profit?



MAXIMUM PROFIT WORKSHEET KEY

1. A grocer sells 50 loaves of bread a day. The cost is \$0.65 a loaf. The grocer estimates that for each \$0.05 price increase, 2 fewer loaves of bread will be sold. What cost will maximize the profit?

Let x = number of \$0.05 price increases.
Thus $0.65 + 0.05x$ represents the single price and
 $50 - 2x$ represents the number of loaves sold.

Income = (number of loaves sold)(loaf price)
Income = $(50 - 2x)(0.65 + 0.05x)$
Income = $32.5 + 1.2x - 0.1x^2$

Notice that the result is a quadratic equation.
The graph of the related function, $y = -0.1x^2 + 1.2x + 32.5$,
opens downward and thus has a maximum point. Since
this is a maximum point, the x -coordinate gives the
number of price increases needed to maximize the profit.

Recall that the x -coordinate of the maximum point
is given by the equation of the axis of symmetry.

$$x = -\frac{b}{2a}$$
$$x = -\frac{1.2}{2(-0.1)}$$
$$x = 6$$

The equation of the axis
of symmetry is $x = 6$.



Axis of symmetry
 $x = 6$

Thus, the profit is maximized when the owner makes six \$0.05 increases.

Single loaf price = $0.65 + 0.05x$

Single loaf price = $0.65 + 0.05(6)$

Single loaf price = \$0.95

Maximum income = (number of loaves sold)(($0.65 + 0.05x$))

Maximum income = $(50 - 2x)(0.65 + 0.05x)$

Maximum income = $(50 - 12)(0.65 + 0.30)$

Maximum income = $(38)(0.95)$

Maximum income = \$36.10

2. A bus company transports 500 people a day between Morse Rd. and high St. The one-way fare is \$0.50. The owners estimates that for each \$0.10 price increase, 50 passengers will be lost. What price will maximize their profit?

Let x = number of \$0.50 price increases.
Thus $0.50 + 0.10x$ represents the single price and
 $500 - 50x$ represents the number of people transported.

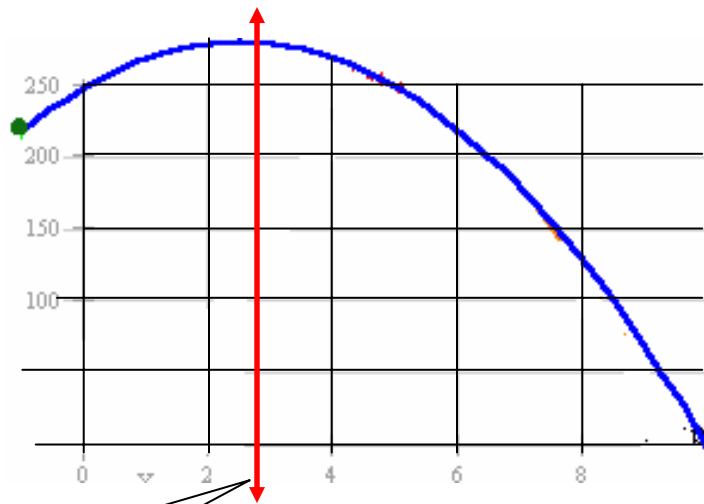
Income = (number of passengers)(ticket price)
Income = $(500 - 50x)(0.50 + 0.10x)$
Income = $250 + 25x - 5x^2$

Notice that the result is a quadratic equation.
The graph of the related function, $y = -5x^2 + 25x + 250$, opens downward and thus has a maximum point. Since this is a maximum point, the x-coordinate gives the number of price increases needed to maximize the profit.

Recall that the x-coordinate of the maximum point is given by the equation of the axis of symmetry.

$$x = -\frac{b}{2a}$$
$$x = -\frac{25}{2(-5)}$$
$$x = 2.5$$

The equation of the axis of symmetry is $x = 2.5$.



Axis symmetry
 $x = 2.5$

Thus, the profit is maximized when the owner makes two and one-half \$0.10 increases.

Single ticket price = $0.50 + 0.10x$

Single ticket price = $0.50 + 0.10(2.5)$

Single ticket price = \$0.75

Maximum income = (number of tickets sold)(($0.50 + 0.10x$))

Maximum income = $(500 - 50x)(0.50 + 0.10x)$

Maximum income = $(500 - 125)(0.50 + 0.25)$

Maximum income = $(375)(0.75)$

Maximum income = \$281.25

Student Name: _____

Date: _____

MAXIMUM PROFIT CHECKLIST

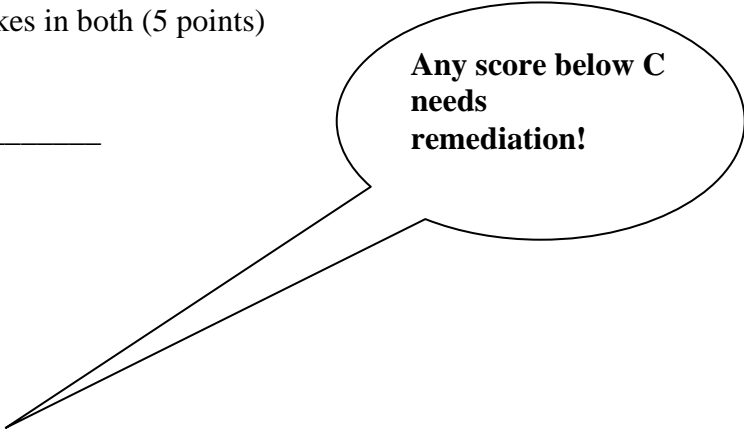
1. On questions (1 – 2), did the student set up a correct quadratic equation?
 - a. Both (30 points)
 - b. One correct and the second with minor mistakes (25 points)
 - c. One correct and the second incorrect (20 points)
 - d. Both incorrect with minor mistakes in each (15 points)
 - e. Both incorrect with minor mistakes in one and major misstates in the second (10 points)
 - f. Major mistakes in both (5 points)

2. On questions (1 – 2), did the student solve the problem correctly?
 - a. Both (30 points)
 - b. One correct and the second with minor mistakes (25 points)
 - c. One correct and the second incorrect (20 points)
 - d. Both incorrect with minor mistakes in each (15 points)
 - e. Both incorrect with minor mistakes in one and major misstates in the second (10 points)
 - f. Major mistakes in both (5 points)

3. On questions (1 – 2), did the student graph correctly?
 - a. Both (30 points)
 - b. One correct and the second with minor mistakes (25 points)
 - c. One correct and the second incorrect (20 points)
 - d. Both incorrect with minor mistakes in each (15 points)
 - e. Both incorrect with minor mistakes in one and major misstates in the second (10 points)
 - f. Major mistakes in both (5 points)

Total Number of Points _____

- A 81 points and above
- B 72 points and above
- C 63 points and above
- D 54 points and above
- F 53 points and below



**Any score below C
needs
remediation!**

