IS YOUR SQUARE COMPLETE? EXAMPLES

1. An equation like "$x^2 - 36 = 0$" can be solved in the following way.

$x^2 - 36 = 0$
$x^2 = 36$
$\sqrt{x^2} = \sqrt{36}$
$|x| = 6$
$x = \pm 6$

Factoring can easily solve this equation. However, its purpose is to show that square roots can be helpful in solving equations.

This same method can be used to solve the equation $x^2 - 4x + 4 = 3$.

2. **Example:** Solve ➔ $x^2 - 4x + 4 = 3$

$x^2 - 4x + 4 = 3$
$(x - 2)^2 = 3$
$\sqrt{(x-2)^2} = \sqrt{3}$
$|x - 2| = \sqrt{3}$
$x - 2 = \pm\sqrt{3}$
$x = 2 \pm \sqrt{3}$

The solution set is $\{2 + \sqrt{3}, 2 - \sqrt{3}\}$

The quadratic expression must be a perfect square in order to use this method. If it is not a perfect square, than a method called "completing the square" may be used.

3. Consider the pattern for squaring a "binomial."

$(x + 6)^2 = x^2 + 2(6)(x) + 6^2$
$= x^2 + 12x + 36$

$(\frac{12}{2}) \rightarrow 6^2$

Notice that "36" is 6^2 and 6 is one-half of 12.

4. To complete the square for an expression of the form "$x^2 + bx$," follow the steps listed below:

1. **Find one-half of "b," the coefficient of "x."**
2. **Square the result of "Step 1."**
3. **Add the result of "Step 2" to "$x^2 + bx$."**

5. **Example:** Find the value of "c" that makes "$x^2 + 14x + c$" a perfect square.

Step 1➔Find one-half of 14.

Step 2➔ Square the result of "Step 1".

Step 2➔ Add the result of "Step 2" to "$x^2 + 14x$.

$\frac{14}{2} = 7$

$7^2 = 49$

$x^2 + 14x + 49$

Thus, c = 49. Notice that $x^2 + 14x + 49$ is equal to $(x + 7)^2$, which is a perfect square.

6. **Example:** Solve➔$x^2 + 6x – 16 = 0$ by completing the square.

$x^2 + 6x – 16 = 0$
$x^2 + 6x = 16$
$x^2 + 6x + 9 = 16 + 9$
$(x + 3)^2 = 25$
$x + 3 = \pm 5$
$x = \pm 5 – 3$
$x = 5 – 3$ or $x = -5 – 3$
$x = 2$ or $x = -8$
The roots of $x^2 + 6x – 16 = 0$ are "2" and "-8".

Notice that $x^2 + 6x – 16$ is not a perfect square. Add 16 to each side. Then complete the square. $(\frac{6}{2})^2 = 9$, so add 9 to each side of the equation.
Factor $x^2 + 6x + 9$. Find the square root of each side. Subtract 3 from each side.

Discuss the above example thoroughly. Make sure students understand the third step of the solution. The number 9 is added because it completes the square for "$x^2 + 6x$." That is, $(\frac{6}{2})^2 = 9$.

You may wish to include the following steps to example # 6:

$$\sqrt{(x+3)^2} = \sqrt{25}$$

$$|x + 3| = 5$$

Check: A calculator may be used to aid in checking the roots of an equation.

7. **Example:** Solve➔ $2x^2 - 9x + 8 = 0$ by completing the square.

$2x^2 - 9x + 8 = 0$

$x^2 - \frac{9}{2}x + 4 = 0$

To complete the square, the coefficient of x^2 must be 1. Divide each side of the equation by 2.

$x^2 - \frac{9}{2}x = -4$

$x^2 - \frac{9}{2}x + \frac{81}{16} = -4 + \frac{81}{16}$

$(x - \frac{9}{4})^2 = \frac{17}{16}$

$x - \frac{9}{4} = \pm\frac{\sqrt{17}}{4}$

$x = \frac{9 \pm \sqrt{17}}{4}$

The roots of $2x^2 - 9x + 8 = 0$ are

$\frac{9+\sqrt{17}}{4}$ **and** $\frac{9-\sqrt{17}}{4}$

Note: If roots involve radicals, they come in conjugate pairs.

Name:_____
Date:_____
Class:_____

IS YOUR SQUARE COMPLETE? WORKSHEET

State whether each trinomial is a perfect square.

1. $x^2 + 8x + 7$
2. $x^2 – 8x + 16$
3. $x^2 + 4x + 3$
4. $x^2 + 12x + 27$
5. $x^2 – 10x + 25$
6. $x^2 – 13x + \frac{169}{4}$

Find the value of "c" that makes each trinomial a perfect square.

7. $x^2 + 8x + c$
8. $x^2 – 6x + c$
9. $x^2 – 7x + c$

Solve each equation by completing the square.

10. $x^2 + 4x + 3 = 0$

11. $x^2 – 4x = 2$

12. $x^2 + 14x – 10 = 5$

13. $4x^2 – 20x + 25 = 0$

IS YOUR SQUARE COMPLETE? WORKSHEET KEY

State whether each trinomial is a perfect square.

1. $x^2 + 8x + 7$ ➔**NO**
2. $x^2 – 8x + 16$ ➔ **YES**
3. $x^2 + 4x + 3$ ➔ **NO**
4. $x^2 + 12x + 27$ ➔ **NO**
5. $x^2 – 10x + 25$ ➔ **YES**
6. $x^2 – 13x + \frac{169}{4}$ ➔**YES**

Find the value of "c" that makes each trinomial a perfect square.

7. $x^2 + 8x + c$ ➔$c = 16$

Step 1➔Find one-half of 8.

Step 2➔ Square the result of "Step 1".

Step 2➔ Add the result of "Step 2" to "$x^2 + 8x$.

$\frac{8}{2} = 4$

$4^2 = 16$

$x^2 + 8x + 16$

8. $x^2 – 6x + c$ ➔ $c = 9$

Step 1➔Find one-half of -6.

Step 2➔ Square the result of "Step 1".

Step 2➔ Add the result of "Step 2" to "$x^2 - 6x$.

$\frac{-6}{2} = -3$

$(-3)^2 = 9$

$x^2 – 6x + 9$

9. $x^2 – 7x + c$

Step 1➔Find one-half of -7.

Step 2➔ Square the result of "Step 1".

Step 2➔ Add the result of "Step 2" to "$x^2 - 7x$.

$\frac{-7}{2}$

$(\frac{-7}{2})^2 = \frac{49}{4}$

$x^2 – 7x + \frac{49}{4}$

Solve each equation by completing the square.

10. $x^2 + 4x + 3 = 0$

$x^2 + 4x = -3$
$x^2 + 4x + (2)^2 = -3 + (2)^2$
$(x + 2)^2 = 1$
$x + 2 = \pm 1$
$x = -1$ or $x = -3$

11. $x^2 - 4x = 2$

$x^2 - 4x + (2)^2 = 2 + (2)^2$
$(x - 2)^2 = 6$
$x - 2 = \pm\sqrt{6}$
$x = 2 \pm\sqrt{6}$

12. $x^2 + 14x - 10 = 5$

$x^2 + 14x = 5 + 10$
$x^2 + 14x + (7)^2 = 15 + (7)^2$
$(x + 7)^2 = 64$
$x + 7 = \pm 8$
$x = 1$ or $x = -15$

13. $4x^2 - 20x + 25 = 0$

$4x^2 - 20x = -25$
$4(x^2 - 5x) = -25$
$4[x^2 - 5x + (\frac{5}{2})^2] = -25 + 25$
$4(x - \frac{5}{2})^2 = 0$
$(x - \frac{5}{2})^2 = 0$
$x = \frac{5}{2}$

Student Name: _____

Date: _____

IS YOUR SQUARE COMPLETE? CHECKLIST

1. On questions (1 – 6), did the student state whether each trinomial is a perfect square correctly?

    a. All six (30 points)
    b. Five of the six (25 points)
    c. Four of the six (20 points)
    d. Three of the six (15 points)
    e. Two of the six (10 points)
    f. One of the six (5 points)

2. On questions (7 – 9), did the student find the value of "c" that makes each trinomial a perfect square?

    a. All three (15 points)
    b. Two of the three (10 points)
    c. One of the three (5 points)

3. On question 10, did the student solve the equation correctly by completing the square?

    a. Yes (15 points)
    b. No, minor mistakes in finding correct terms to make perfect square (10 points)
    c. No, minor mistakes in solving equation (5 points)

4. On question 11, did the student solve the equation correctly by completing the square?

    a. Yes (15 points)
    b. No, minor mistakes in finding correct terms to make perfect square (10 points)
    c. No, minor mistakes in solving equation (5 points)

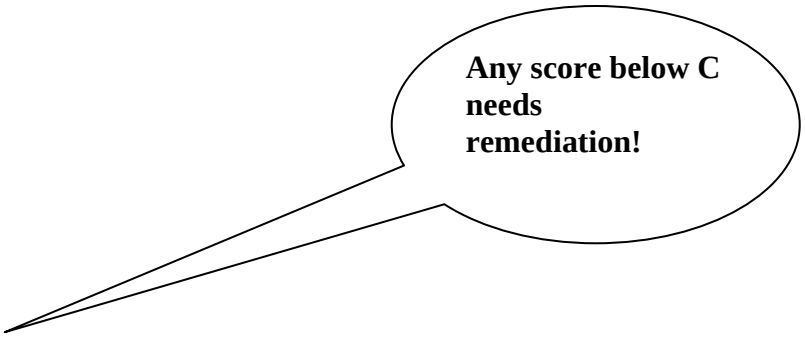
5. On question 12, did the student solve the equation correctly by completing the square?

    a. Yes (15 points)
    b. No, minor mistakes in finding correct terms to make perfect square (10 points)
    c. No, minor mistakes in solving equation (5 points)

6. On question 13, did the student solve the equation correctly by completing the square?

    a. Yes (15 points)
    b. No, minor mistakes in finding correct terms to make perfect square (10 points)
    c. No, minor mistakes in solving equation (5 points)

Total Number of Points _____



A 94 points and above

B 84 points and above

C 73 points and above

D 63 points and above

F 62 points and below