

How Simple Is Your Rational Expression? Examples

1. Rational expressions are algebraic expressions whose numerator and denominator are polynomials. The expression $\frac{5x+3}{y}$, $\frac{2}{x}$, and $\frac{a-2}{a^2+4}$ are examples of rational expressions or algebraic fractions.
2. A fraction indicates division. Zero cannot be used as a denominator because division by zero is undefined.
3. Therefore, any value assigned to a variable that results in a denominator of zero must be excluded from the domain of the variable (it may be necessary to review factoring.)
4. For example, the following would have exclusions:

$$\frac{5}{x}, \text{ would exclude } x = 0.$$

$$\frac{3x+7}{x+4}, \text{ would exclude } x = -4.$$

$$\frac{y^2-5}{x^2-5x+6}, \text{ would exclude } x = 2 \text{ and } x = 3.$$

Emphasize that $x \neq 4$ because $-4 + 4 = 0$.

**$x^2 - 5x + 6$ factored is $(x - 2)(x - 3)$.
Apply the Zero Product Property to show that $x \neq 2$ or $x \neq 3$.**

5. **Example:** State the value(s) of the variable that must be excluded:

$$\frac{5x}{x+7}$$

Exclude the values for which $x + 7 = 0$.
 $x + 7 = 0$
 $x = -7$

6. **Example:** State the values of the variable that must be excluded:

$\frac{2a-3}{a^2-a-12}$	<div style="border: 1px solid black; border-radius: 15px; padding: 10px; width: fit-content;"> <p>Factor $a^2 - a - 12 = 0$.</p> <p>Use the Zero Product Property.</p> </div>
<p>Exclude the values for which $a^2 - a - 12 = 0$.</p>	
<p>$a^2 - a - 12 = 0$</p>	
<p>$(a - 4)(a + 3) = 0$</p>	
<p>$a = 4$ or $a = -3$</p>	
<p>Therefore, "a" cannot equal 4 or -3.</p>	

7. Recall that to simplify an algebraic fraction such as $\frac{14a^2bc}{42abc^2}$, first factor the numerator and denominator. Then simplify common factors.

$\frac{14a^2bc}{42abc^2} \rightarrow \frac{2 \cdot 7 \cdot a \cdot a \cdot b \cdot c}{2 \cdot 3 \cdot 7 \cdot a \cdot b \cdot c \cdot c} \rightarrow \frac{a}{3c}$	<div style="border: 1px solid black; border-radius: 15px; padding: 10px; width: fit-content;"> <p>Notice that $a \neq 0$, $b \neq 0$, and $c \neq 0$.</p> </div> <div style="border: 1px solid black; border-radius: 15px; padding: 10px; width: fit-content; margin-top: 10px;"> <p>The GCF is $14abc$</p> </div>

This same procedure can be used to simplify rational expressions having polynomials in the numerator and denominator.

8. **Example:** Simplify $\rightarrow \frac{3a^2 + a - 2}{a^2 + 7a + 6}$. State the excluded values of "a."

$$\frac{3a^2 + a - 2}{a^2 + 7a + 6} = \frac{(a+1)(3a-2)}{(a+6)(a+1)} = \frac{(a+1)(3a-2)}{(a+6)(a+1)}$$
$$= \frac{3a-2}{a+6}$$

The excluded values of "a" are any values for which $a^2 + 7a + 6 = 0 \rightarrow (a+6)(a+1) = 0 \rightarrow a = -6$ or $a = -1$.

Therefore, a cannot equal -6 or -1.

Stress that factors must be reduce, not terms.

9. **Example:** Simplify $\rightarrow \frac{2x-2y}{y^2-x^2}$. State the excluded values of "x" and "y."

$$\frac{2x-2y}{y^2-x^2} = \frac{2(x-y)}{(y-x)(y+x)} = \frac{2(x-y)}{(y-x)(y+x)}$$
$$= \frac{-2}{y+x} \text{ or } -\frac{2}{y+x}$$

The excluded values of "x" and "y" are any values for which $y^2 - x^2 = 0 \rightarrow (y-x)(y+x) = 0 \rightarrow y = x$ or $y = -x$

Therefore, y cannot equal x or -x.

Another way to state the excluded values is x cannot equal y or -y.

Name: _____

Date: _____

Class: _____

How Simple Is Your Rational Expression? Worksheet

Simplify each rational expression. State the excluded values of the variables.

1. $\frac{y-3}{y^2-9}$

6. $\frac{4x+8}{x^2+6x+8}$

2. $\frac{r^3-r^2}{r-1}$

7. $\frac{9-a^2}{a^2-a-6}$

3. $\frac{3m^2}{6m^2-3m}$

8. $\frac{g^2+g-2}{g^2-3g+2}$

4. $\frac{c-6}{c^2-12c+36}$

9. $\frac{a^2-9}{a^2+6a-27}$

5. $\frac{m^2-36}{m^2+5m-6}$

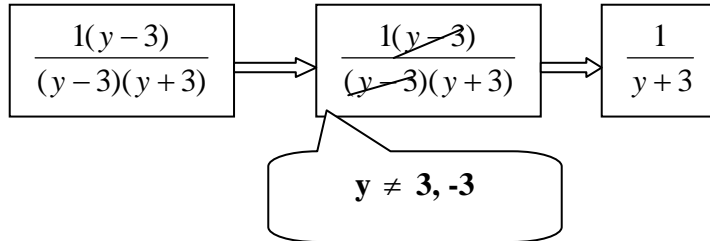
10. $\frac{k^2-1}{k^2+2k+1}$



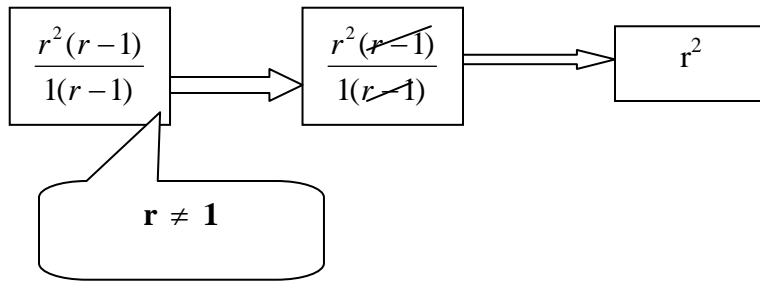
How Simple Is Your Rational Expression? Worksheet Key

Simplify each rational expression. State the excluded values of the variables.

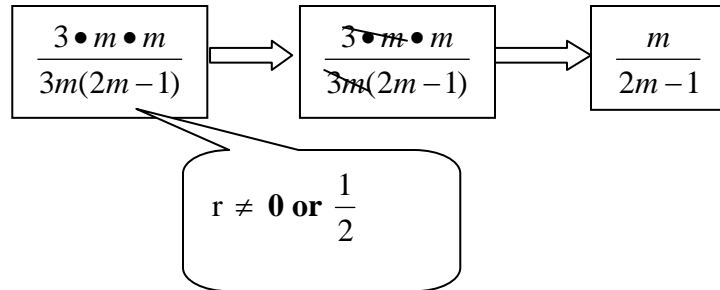
1. $\frac{y-3}{y^2-9}$



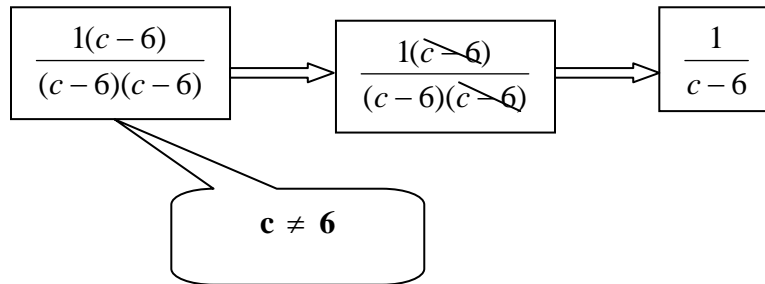
2. $\frac{r^3-r^2}{r-1}$



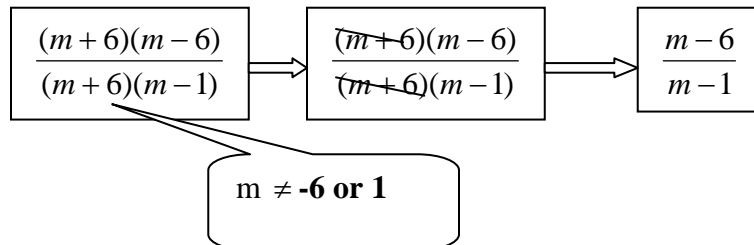
3. $\frac{3m^2}{6m^2-3m}$



4. $\frac{c-6}{c^2-12c+36}$



5. $\frac{m^2-36}{m^2+5m-6}$



6. $\frac{4x+8}{x^2+6x+8}$

$$\frac{4(x+2)}{(x+2)(x+4)} \Rightarrow \frac{4\cancel{(x+2)}}{\cancel{(x+2)}(x+4)} \Rightarrow \frac{4}{x+4}$$

$x \neq -2$ or -4

7. $\frac{9-a^2}{a^2-a-6}$

$$\frac{(3-a)(3+a)}{(a-3)(a+2)} \Rightarrow \frac{-(a-3)(a+3)}{(a-3)(a+2)} \Rightarrow -\frac{a+3}{a+2}$$

$a \neq 3, -2$

8. $\frac{g^2+g-2}{g^2-3g+2}$

$$\frac{(g+2)(g-1)}{(g-2)(g-1)} \Rightarrow \frac{(g+2)\cancel{(g-1)}}{(g-2)\cancel{(g-1)}} \Rightarrow \frac{g+2}{g-2}$$

$g \neq 2$ or 1

9. $\frac{a^2-9}{a^2+6a-27}$

$$\frac{(a-3)(a+3)}{(a-3)(a+9)} \Rightarrow \frac{\cancel{(a-3)}(a+3)}{\cancel{(a-3)}(a+9)} \Rightarrow \frac{a+3}{a+9}$$

$a \neq 3$ or -9

10. $\frac{k^2-1}{k^2+2k+1}$

$$\frac{(k-1)(k+1)}{(k+1)(k+1)} \Rightarrow \frac{(k-1)\cancel{(k+1)}}{(k+1)\cancel{(k+1)}} \Rightarrow \frac{k-1}{k+1}$$

$k \neq -1$

Student Name: _____

Date: _____

How Simple Is Your Rational Expression? Checklist

1. On question 1, did the student simplify the rational expression completely and state the excluded values?
 - a. Yes (20 points)
 - b. Simplified completely but did not state excluded values (15 points)
 - c. Did not simplify completely but did state excluded values (10 points)
 - d. Did not simplify completely and did not state excluded values (5 points)

2. On question 2, did the student simplify the rational expression completely and state the excluded values?
 - a. Yes (20 points)
 - b. Simplified completely but did not state excluded values (15 points)
 - c. Did not simplify completely but did state excluded values (10 points)
 - d. Did not simplify completely and did not state excluded values (5 points)

3. On question 3, did the student simplify the rational expression completely and state the excluded values?
 - a. Yes (20 points)
 - b. Simplified completely but did not state excluded values (15 points)
 - c. Did not simplify completely but did state excluded values (10 points)
 - d. Did not simplify completely and did not state excluded values (5 points)

4. On question 4, did the student simplify the rational expression completely and state the excluded values?
 - a. Yes (20 points)
 - b. Simplified completely but did not state excluded values (15 points)
 - c. Did not simplify completely but did state excluded values (10 points)
 - d. Did not simplify completely and did not state excluded values (5 points)

5. On question 5, did the student simplify the rational expression completely and state the excluded values?
 - a. Yes (20 points)
 - b. Simplified completely but did not state excluded values (15 points)
 - c. Did not simplify completely but did state excluded values (10 points)
 - d. Did not simplify completely and did not state excluded values (5 points)

6. On question 6, did the student simplify the rational expression completely and state the excluded values?
 - a. Yes (20 points)
 - b. Simplified completely but did not state excluded values (15 points)
 - c. Did not simplify completely but did state excluded values (10 points)
 - d. Did not simplify completely and did not state excluded values (5 points)

7. On question 7, did the student simplify the rational expression completely and state the excluded values?
 - a. Yes (20 points)
 - b. Simplified completely but did not state excluded values (15 points)
 - c. Did not simplify completely but did state excluded values (10 points)
 - d. Did not simplify completely and did not state excluded values (5 points)

8. On question 8, did the student simplify the rational expression completely and state the excluded values?
 - a. Yes (20 points)
 - b. Simplified completely but did not state excluded values (15 points)
 - c. Did not simplify completely but did state excluded values (10 points)
 - d. Did not simplify completely and did not state excluded values (5 points)

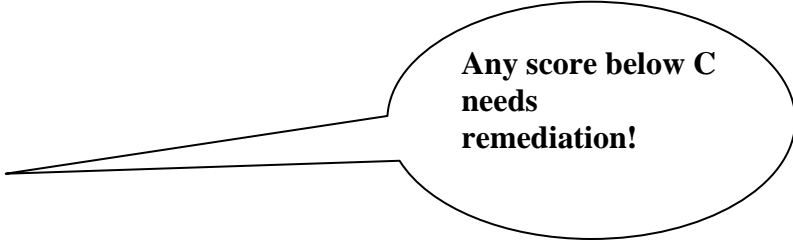
9. On question 9, did the student simplify the rational expression completely and state the excluded values?
 - a. Yes (20 points)
 - b. Simplified completely but did not state excluded values (15 points)
 - c. Did not simplify completely but did state excluded values (10 points)
 - d. Did not simplify completely and did not state excluded values (5 points)

10. On question 10, did the student simplify the rational expression completely and state the excluded values?
 - a. Yes (20 points)
 - b. Simplified completely but did not state excluded values (15 points)
 - c. Did not simplify completely but did state excluded values (10 points)
 - d. Did not simplify completely and did not state excluded values (5 points)

 - e. No, student identified a cluster of data (10 points)

Total Number of Points _____

- A 180 points and above
- B 160 points and above
- C 140 points and above
- D 120 points and above
- F 119 points and below



**Any score below C
needs
remediation!**