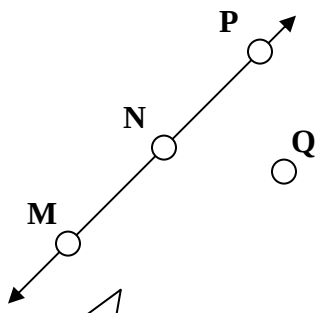
Finding the Measure of Segments Examples

1. In geometry, the distance between two points is used to define the measure of a segment. Segments can be defined by using the idea of betweenness.

In the figure at the right, point N is **between** M and P while point Q is not between M and P. For N to be between M and P, all three points must be colinear. **Segment MP**, $\overline{MP}$, consists of points M and P and all points between M and P. The **measure** of $\overline{MP}$, written MP (without a bar over the letter), is the distance between M and P. Thus, the measure of a segment is the same as the distance between its two endpoints.

In order to quantify the measure of a segment, you must measure the segment using a device, like a ruler, that has a **unit of measure**, such as inches.

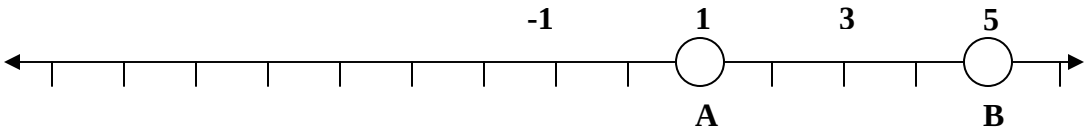


When discussing the figure, point out than $\overline{NP}$ is not the same as $\overline{MP}$, while the line defined by M and P is the same as the line defined by N and P.

2. **Ruler Postulate** – The points on any line can be paired with the real numbers so that, given any two points X and Y on the line, X corresponds to zero, and Y corresponds to a positive number.

Remind students that a postulate is a statement that is assumed to be true.

3. The distance between points may be measured using a number line.



To find the measure of $\overline{AB}$, you first need to identify the coordinates of A and B. The coordinate of A is 1 and the coordinate of B is 5. Since measure is always a positive number, you can subtract the lesser coordinate from the greater one, or find the absolute value of the difference. When you use absolute value, the order in which you subtract the coordinates does not matter.

Distance from A to B

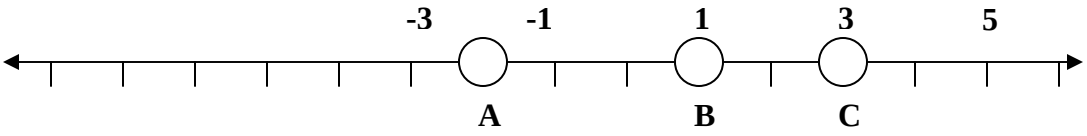
$|5 - 1| = |4| = 4$

Distance from B to A

$|1 - 5| = |-4| = 4$

Finding the distance from A to B or from B to A results in the same measure.

4. **Example** – Find AB, BC, and AC on the number line shown below:



AB = |–2 – 1|
AB = |-3|
AB = 3

BC = |1 – 3|
BC = |-2|
BC = 2

AC = |–2 – 3|
AC = |-5|
AC = 5

In this example, B is between A and C and AB + BC = AC (3 + 2 = 5). This example leads to the following postulate.

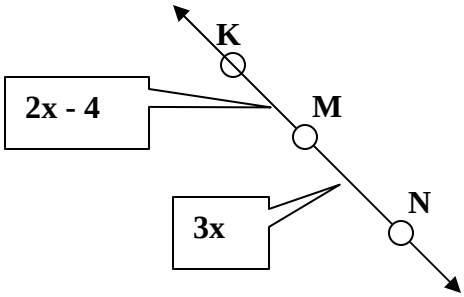
5. **Segment Addition Postulate** – If Q is between P and R, then PQ + QR = PR. If PQ + QR = PR then Q is between P and R.

6. **Example** – Find the measure of $\overline{MN}$ if M is between K and N, KM = 2x – 4, MN = 3x, and KN = 26.

Since M is between K and N, KM + MN = KN.

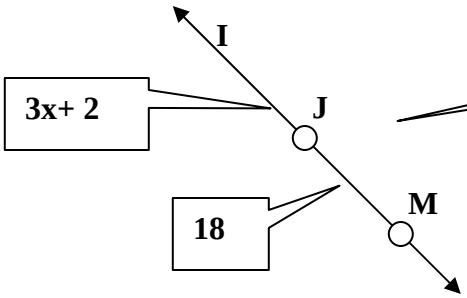
KM + MN = KN
(2x – 4) + 3x = 26
5x – 4 = 26
5x = 30
x = 6

MN = 3x
MN = 3(6)
$\overline{MN}$ **= 18**



7. **Example** – Find the measure of $\overline{IJ}$ if J is between I and M, IJ = 3x + 2, JM = 18, and IM = 5x.

First draw the line segment and label its points.



IM = IJ + JM

5x = (3x + 2) +(18)
5x = 3x + 20
2x = 20
x = 10

IJ = 3x + 2 ➔ IJ = 3(10) + 2 ➔ IJ = 32

8. The distance, d, between any points with coordinates (x_1, y_1) and (x_2, y_2) can be found by using the distance formula.

Distance Formula	**d =** $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

The distance between points is the same no matter which point is called (x_1, y_1).

9. Find the length of the segment with endpoints A(–2, 3) and B(5, –3).

Let (–2, 3) be (x_1, y_1) and (5, –3) be (x_2, y_2).

D = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ➔ $\sqrt{(5-(-2))^2 + (-3-3)^2}$

D = $\sqrt{7^2 + (-6)^2}$ ==+ $\sqrt{85}$

D $\cong$ **9.22**

The length of $\overline{AB}$ **is about 9.22 units.**

When discussing the distance between two points, make sure students understand that the measure is always positive. Ask your students if they could draw a segment of –3 inches, or if they could measure –1 cup of water.

Emphasize to students that AB represents a number and $\overline{AB}$ **represents a segment.**

10. Find the distance between points W(1, 2) and Z(–4, –2).

Let (1, 2) be (x_1, y_1) and (–4, –2) be (x_2, y_2).

D = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ➔ $\sqrt{(-4-(1))^2 + (-2-2)^2}$

D = $\sqrt{(-5)^2 + (-4)^2}$ ➔ $\sqrt{41}$

D $\cong$ **6.403**

The length of $\overline{WZ}$ **is about 6.403 units.**

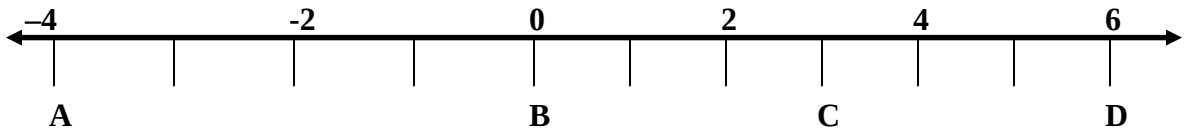
Name:_____
Date:_____
Class:_____

Finding the Measure of Segments Worksheet

1. When using the distance formula to find the distance between points A(18, 8) and B(5, 7), do you have to choose 18 for x_1? Explain.

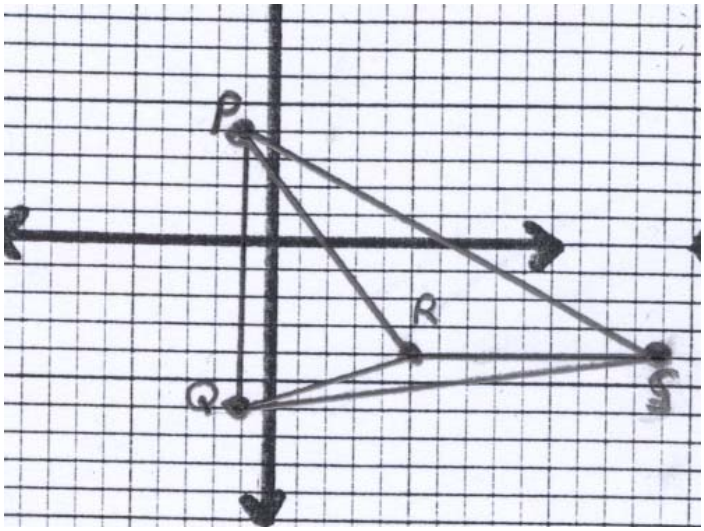
2. Draw $\overline{AB}$ and $\overline{BC}$ such that AB + BC ≠ AC.

For questions 3-8, refer to the number line below to find each measure.



3. AB
4. CD
5. BD
6. CB
7. DA
8. AC

Refer to the coordinate plane at the right to find each measure. If the measure is not a whole number, round the result to the nearest hundredth.



9. PQ
10. SR
11. RP
12. PS
13. QR
14. QS

Note: Find the coordinates of each point before finding the measure.

15. If B is between A and C, AB = x, BC = 2x + 1, and AC = 22, find the value of x and the measure of $\overline{BC}$.

Given that J is between H and K, find each missing measure.

16. HJ = 17, JK = 6, HK = ____

17. HJ = $2\sqrt{2}$, JK = $3\sqrt{2}$, HK = ____

18. HJ = 23.7, JK = ____, HK = 35.2

19. HJ = ____, JK = $2\frac{1}{2}$, HK = $6\frac{2}{5}$

If B is between A and C, find the value of x and the measure of $\overline{BC}$.

20. AB = 3, BC = 4x + 1, AC = 8

21. AB = x + 2, BC = 2x – 6, AC = 20

22. AB = 24, BC = 3x, AC = 7x – 4

23. AB = 3, BC = 2x + 5, AC = 11x + 2

24. Find the perimeter of the triangle with vertices X(2, -1), Y(5, 3), and Z(-3, 11). Round your result to the nearest hundredth.

25. Find the value of "a" so that the distance between points A(4, 7) and B(a, 3) is 5 units.

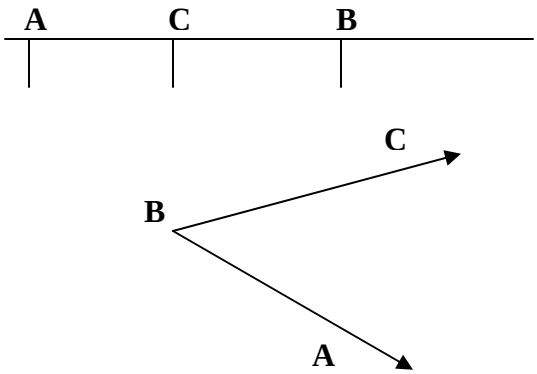
Finding the Measure of Segments Worksheet Key

1. When using the distance formula to find the distance between points A(18, 8) and B(5, 7), do you have to choose 18 for x_1? Explain.

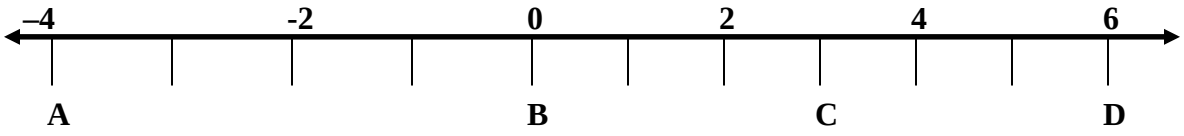
No. You can use 18 for x_2 as long as you use 8 for y_2, 5 for x_1, and 7 for y_1.

2. Draw $\overline{AB}$ and $\overline{BC}$ such that AB + BC ≠ AC.

Answers will vary. Sample answers:



For questions 3-8, refer to the number line below to find each measure.



3. AB ➔ **|-4 – 0| = 4**

4. CD ➔ **|3 – 6| = 3**

5. BD ➔ **|0 – 6| = 6**

6. CB ➔ **|3 – 0| = 3**

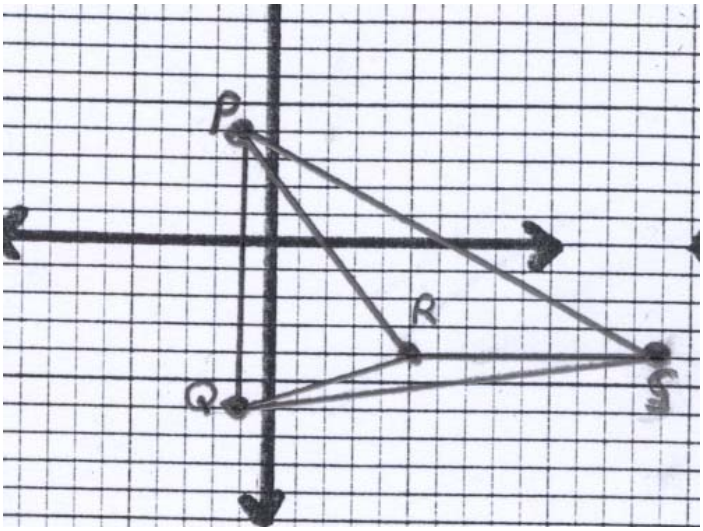
7. DA ➔ **|6 - -4| = 10**

8. AC ➔ **|-4 – 3| = 7**

Refer to the coordinate plane at the right to find each measure. If the measure is not a whole number, round the result to the nearest hundredth.

P(-1, 4) Q(-1, -6)

R(5, -4) S(14, -4)



9. PQ

Let (–1, 4) be (x_1, y_1) and (–1, –6) be (x_2, y_2).

D = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ➔ $\sqrt{(-1-(-1))^2 + (-6-4)^2}$

D = $\sqrt{(0)^2 + (-10)^2}$ ➔ **10**

D = 10

The length of $\overline{PQ}$ **is 10 units.**

10. SR

Let (14, -4) be (x_1, y_1) and (5, –4) be (x_2, y_2).

D = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ➔ $\sqrt{(5-(14))^2 + (-4-(-4)^2}$

D = $\sqrt{(-9)^2 + (0)^2}$ ➔ **9**

D = 9

The length of $\overline{SR}$ **is 9 units.**

11. RP

Let (5, -4) be (x_1, y_1) and (–1, 4) be (x_2, y_2).

D = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ➔ $\sqrt{(-1-(5))^2 + (4-(-4)^2}$

D = $\sqrt{(-6)^2 + (8)^2}$ ➔ **10**

D = 10

The length of $\overline{RP}$ **is 10 units.**

12. PS

Let (-1, 4) be (x_1, y_1) and (14, -4) be (x_2, y_2).

D = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ➔ $\sqrt{(14-(-1))^2 + (-4-(4)^2}$

D = $\sqrt{(15)^2 + (-8)^2}$ ➔ **17**

D = 17

The length of $\overline{PS}$ **is 17 units.**

13. QR

Let (-1, -6) be (x_1, y_1) and (5, -4) be (x_2, y_2).

D = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ➔ $\sqrt{(5-(-1))^2 + (-4-(-6)^2}$

D = $\sqrt{(6)^2 + (2)^2}$ ➔ $\cong$ **6.32**

D $\cong$ **6.32**

The length of $\overline{QR}$ **is 6.32 units.**

14. QS

Let (-1, -6) be (x_1, y_1) and (14, -4) be (x_2, y_2).

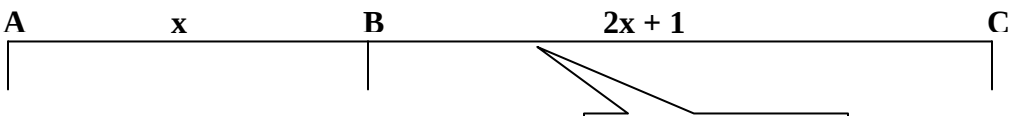
D = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ➔ $\sqrt{(14 - (-1))^2 + (-4 - (-6))^2}$

D = $\sqrt{(15)^2 + (2)^2}$ ➔ **≅ 15.13**

D ≅ 15.13

The length of $\overline{QS}$ is 15.13 units.

15. If B is between A and C, AB = x, BC = 2x + 1, and AC = 22, find the value of x and the measure of $\overline{BC}$.



BC = 2(7) + 1

BC = 15

AB + BC = AC

(x) + (2x + 1) = 22

3x + 1 = 22

3x = 21

x = 7

Given that J is between H and K, find each missing measure.

16. HJ = 17, JK = 6, HK = ____

HJ + JK = HK
17 + 6 = HK
HK = 23

17. HJ = $2\sqrt{2}$, JK = $3\sqrt{2}$, HK = ____

HJ + JK = HK
$2\sqrt{2}$ + $3\sqrt{2}$ = HK
HK = $5\sqrt{2}$

18. HJ = 23.7, JK = ____, HK = 35.2

HJ + JK = HK
23.7 + JK = 35.2
JK = 11.5

19. HJ = ____, JK = $2\frac{1}{2}$, HK = $6\frac{2}{5}$

HJ + JK = HK
HJ + $2\frac{1}{2}$ **=** $6\frac{2}{5}$
HJ = $3\frac{9}{10}$

If B is between A and C, find the value of x and the measure of $\overline{BC}$.

20. AB = 3, BC = 4x + 1, AC = 8

AB + BC = AC

(3) + (4x + 1) = 8

4x + 4 = 8

4x = 4

x = 1

BC = 4x + 1
BC = 4(1) + 1
BC = 5

21. AB = x + 2, BC = 2x – 6, AC = 20

AB + BC = AC

(x + 2) + (2x - 6) = 20

3x – 4 = 20

3x = 24

x = 8

BC = 2x - 6
BC = 2(8) - 6
BC = 10

22. AB = 24, BC = 3x, AC = 7x – 4

AB + BC = AC

(24) + (3x) = 7x - 4

24 = 4x - 4

28 = 4x

x = 7

BC = 3x
BC = 3(7)
BC = 21

23. AB = 3, BC = 2x + 5, AC = 11x + 2

AB + BC = AC

(3) + (2x + 5) = 11x + 2

6 = 9x

$\frac{2}{3}$ **= x**

BC = 2x + 5

BC = $2(\frac{2}{3}) + 5$

BC = $\frac{19}{3}$

24. Find the perimeter of the triangle with vertices X(2, -1), Y(5, 3), and Z(-3, 11). Round your result to the nearest hundredth.

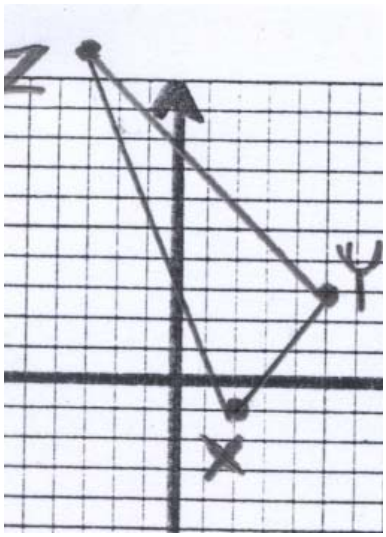
Perimeter = XY + YZ + ZX

XY = $\sqrt{(5-2)^2 + (3-(-1))^2}$
XY = 5

YZ = $\sqrt{(-3-5)^2 + (11-3)^2}$
YZ = 11.31

ZX = $\sqrt{(2-(-3))^2 + (-1-11)^2}$
ZX = 13

Perimeter = 5 + 11.31 + 13 ➔29.31



25. Find the value of "a" so that the distance between points A(4, 7) and B(a, 3) is 5 units.

AB = $\sqrt{(a-4)^2+(3-7)^2}$

5 = $\sqrt{(a-4)^2+(-4)^2}$

$25 = (a-4)^2 + (-4)^2$

$25 = (a-4)^2 + 16$

$9 = (a-4)^2$

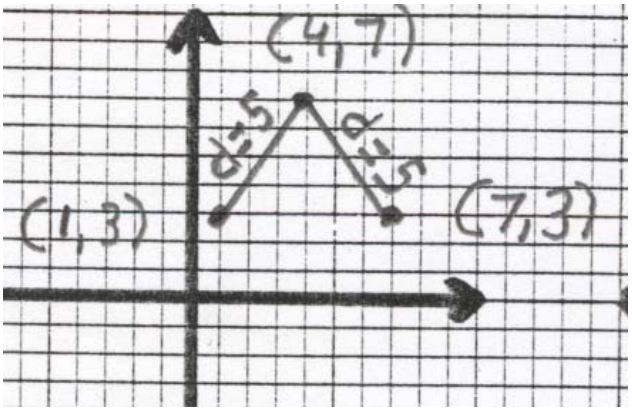
$\sqrt{9} = \sqrt{(a-4)^2}$

$\pm 3 = a - 4$

$4 \pm 3 = a$

a = 7 or 1

B(7, 3) or B(1, 3)



Student Name: _____

Date: _____

Finding the Measure of Segments Checklist

1. On question 1, did the student answer all parts correctly?

    a. Yes (15 points)
    b. Student answered correctly but explanation was not correct (10 points)
    c. Student answered correctly but no explanation (5 points)

2. On question 2, did the student draw the examples correctly?

    a. Yes (5 points)

3. On questions 3 through 8, did the student find the measure of each segment correctly?

    a. All six (30 points)
    b. Five of the six (25 points)
    c. Four of the six (20 points)
    d. Three of the six (15 points)
    e. Two of the six (10 points)
    f. One of the six (5 points)

4. On questions 9 through 12, did the student find the measure of each segment correctly?

    a. All four (20 points)
    b. Three of the four (15 points)
    c. Two of the four (10 points)
    d. One of the four (5 points)

5. On questions 13 and 14, did the student find the measure of each segment and round off correctly?

    a. Yes (15 points)
    b. Found the measure on both segments but did not round off correctly on one of the two. (10 points)
    c. Found the measure on both segments but did not round off correctly on either segment. (5 points)
    d. Found the measure on one of the segments and rounded it off correctly. (5 points)

6. On question 15, did the student find the value of x and the measure of $\overline{BC}$?

    a. Yes (10 points)
    b. Found the value of x but the measure of $\overline{BC}$ is incorrect. (5 points)

7. On questions 16 through 19, did the student find the missing measure?

    a. All four (20 points)
    b. Three of the four (15 points)
    c. Two of the four (10 points)
    d. One of the four (5 points)

8. On question 20, did the student find the value of x and the measure of $\overline{BC}$?

    a. Yes (10 points)
    b. Found the value of x but the measure of $\overline{BC}$ is incorrect. (5 points)

9. On question 21, did the student find the value of x and the measure of $\overline{BC}$?

    a. Yes (10 points)
    b. Found the value of x but the measure of $\overline{BC}$ is incorrect. (5 points)

10. On question 22, did the student find the value of x and the measure of $\overline{BC}$?

    a. Yes (10 points)
    b. Found the value of x but the measure of $\overline{BC}$ is incorrect. (5 points)

11. On question 23, did the student find the value of x and the measure of $\overline{BC}$?

    a. Yes (10 points)
    b. Found the value of x but the measure of $\overline{BC}$ is incorrect. (5 points)

12. On question 24, did the student find the perimeter of triangle XYZ correctly?

    a. Yes (15 points)
    b. No, but distance on two of the three sides was found correctly. (10 points)
    c. No, but distance on one of the three sides was found correctly. (5 points)

13. On question 25, did the student find the correct values of "a"?

    a. Both (10 points)
    b. One of the two (5 points)

Total Number of Points _____

Any score below C needs remediation!

A	162 points and above
B	144 points and above
C	126 points and above
D	108 points and above
F	107 points and below