Angles and Algebra Examples

1. A protractor can be used to measure angles as shown below.

2. \( \angle ABC \) measures 60°. \( \angle DBC \) measures 120°. What is the sum of the degree measures of \( \angle ABC \) and \( \angle DBC \)? \( 60^\circ + 120^\circ = 180^\circ \).

**Definition of Supplementary Angles**
Two angles are supplementary if the sum of their degree measures is 180.

3. Review with students how to name angles.

**Name each angle in two ways.**

1. \( \angle XYZ \)
   \( \angle Y \)

2. \( \angle RST \)
   \( \angle S \)
4. **Example** – The measure of an angle is three times its supplement. Find the measure of each angle.

\[
x + 3x = 180
\]
\[
4x = 180
\]
\[
x = 45
\]

Let \( x \) be the measure of the lesser angle and \( 3x \) be the measure of the greater angle.

The measures are 45° and \( 3(45°) \), or 135°.

5. **Definition of Complementary Angles**

Two angles are complementary if the sum of their degree measures is 90.

6. **EXAMPLE** – An angle is 16° greater than its complement. Find the measure of each angle.

\[
x + (x + 16) = 90
\]
\[
2x + 16 = 90
\]
\[
x = 37
\]

Let \( x \) be the measure of the lesser angle and \( (x + 16) \) the measure of the greater angle.

The measures are 37° and \( (37° + 16°) \), or 53°.

7. **EXAMPLE** – The measure of an angle is 20° less than three times its supplement. Find the measure of each angle.

\[
x + (3x - 20) = 180
\]
\[
4x = 200
\]
\[
x = 50
\]

Let \( x \) be the measure of the lesser angle and \( 3x - 20 \) be the measure of the greater angle.

The measures are 50° and \( 3(50°) - 20° \), or 130°.
8. **Example** – An angle measures 42° less than its complement. Find the measure of each angle.

\[ x + (x - 42) = 90 \]
\[ 2x - 42 = 90 \]
\[ 2x = 132 \]
\[ x = 66 \]

The measures are 66° and (66° - 42°), or 24°.

9. **Thought Provoker** – *What is the sum of the degree measures of the three angles of a triangle?* Give the students protractors and have them measure the triangles below. Then have students compile their data and make a generalization.

The sum of the angles equals 180°.
10. **Example** – What are the measures of the angles of an equilateral triangle?

In an equilateral triangle, the sides are congruent and the angles are congruent.

\[(x) + (x) + (x) = 180\]
\[3x = 180\]
\[x = 60\]

Each angle measures 60°.

11. **Example** – What are the measures of the angles of an isosceles right triangle?

An isosceles right triangle contains a right angle and two congruent angles.

\[(x) + (x) + 90 = 180\]
\[2x = 90\]
\[x = 45\]

The measures are 45°, 45°, and 90°.
Angles and Algebra Worksheet

Find the complement of each angle measure.

1. 85°
2. 42°
3. 13°
4. 45°
5. x°
6. 3x°
7. (2x + 40)°
8. (x – 7)°

Find the supplement of each angle measure.

9. 130°
10. 65°
11. 127°
12. 87°
13. y°
14. 6m°
15. (3x + 5)°
16. (x – 20)°

Find the measure of the third angle of each triangle in which the measures of two angles of the triangle are given.

17. 16°, 42°
18. 40°, 70°
19. 50°, 45°
20. 90°, 30°
21. x°, y°
22. x°, (x + 20)°
23. y°, (y – 10)°
24. m°, (2m + 1)°
25. One of the congruent angles of an isosceles triangle measures 37°. Find the measures of the other angles.

26. The measures of the angles of a certain triangle are consecutive even integers. Find their measures.

27. An angle measures 38° less than its complement. Find the measures of the two angles.

28. One angle of a triangle measures 10° more than the second. The measure of the third angle is twice the sum of the first two angles. Find the measure of each angle.

29. One of two complementary angles measures 30° more than three times the other. Find the measure of each angle.

30. Find the measure of an angle that is 10° more than its complement.

31. Find the measure of an angle that is 30° less than its supplement.

32. Find the measure of an angle that is one-half the measure of its complement.

33. Find the measure of an angle that is one-half the measure of its supplement.
Angles and Algebra Worksheet Key

Find the complement of each angle measure.

1. $85^\circ \rightarrow 90 - 85 = 5^\circ$
2. $42^\circ \rightarrow 90 - 42 = 48^\circ$
3. $13^\circ \rightarrow 90 - 13 = 77^\circ$
4. $45^\circ \rightarrow 90 - 45 = 45^\circ$
5. $x^\circ \rightarrow (90 - x)^\circ$
6. $3x^\circ \rightarrow (90 - 3x)^\circ$
7. $(2x + 40)^\circ \rightarrow 90 - (2x + 40)^\circ \rightarrow \quad (50 - 2x)^\circ$
8. $(x - 7)^\circ \rightarrow 90 - (x - 7)^\circ \rightarrow \quad (97 - x)^\circ$

Find the supplement of each angle measure.

9. $130^\circ \rightarrow 180 - 130 = 50^\circ$
10. $65^\circ \rightarrow 180 - 65 = 115^\circ$
11. $127^\circ \rightarrow 180 - 127 = 53^\circ$
12. $87^\circ \rightarrow 180 - 87 = 93^\circ$
13. $y^\circ \rightarrow (180 - y)^\circ$
14. $6m^\circ \rightarrow (180 - 6m)^\circ$
15. $(3x + 5)^\circ \rightarrow 180 - (3x + 5)^\circ \rightarrow \quad (175 - 3x)^\circ$
16. $(x - 20)^\circ \rightarrow 180 - (x - 20)^\circ \rightarrow \quad (200 - x)^\circ$

Find the measure of the third angle of each triangle in which the measures of two angles of the triangle are given.

17. $16^\circ, 42^\circ \rightarrow 180 - (16 + 42) = 122^\circ$
18. $40^\circ, 70^\circ \rightarrow 180 - (40 + 70) = 70^\circ$
19. $50^\circ, 45^\circ \rightarrow 180 - (50 + 45) = 85^\circ$
20. $90^\circ, 30^\circ \rightarrow 180 - (90 + 30) = 60^\circ$
21. $x^\circ, y^\circ \rightarrow (180 - x - y)^\circ$
22. $x^\circ, (x + 20)^\circ \rightarrow 180 - \quad (2x + 20)^\circ \rightarrow (160 - 2x)^\circ$
23. $y^\circ, (y - 10)^\circ \rightarrow 180 - \quad (2y - 10)^\circ \rightarrow (190 - 2y)^\circ$
24. $m^\circ, (2m + 1)^\circ \rightarrow 180 - \quad (3m + 1)^\circ \rightarrow (179 - 3m)^\circ$
25. One of the congruent angles of an isosceles triangle measures 37°. Find the measures of the other angles.

\[
x + 37 + 37 = 180
\]
\[
x + 74 = 180
\]
\[
x = 106
\]

Angles are 106°, 37°, and 37°.

26. The measures of the angles of a certain triangle are consecutive even integers. Find their measures.

\[
(x) + (x + 2) + (x + 4) = 180
\]
\[
3x + 6 = 180
\]
\[
3x = 174
\]
\[
x = 58 \quad \text{The angles are 58°, 60°, and 62°.}
\]

27. An angle measures 38° less than its complement. Find the measures of the two angles.

\[
(x) + (x - 38) = 90
\]
\[
2x - 38 = 90
\]
\[
2x = 128
\]
\[
x = 64
\]

The angles are 26°, and 64°.
28. One angle of a triangle measures 10° more than the second. The measure of the third angle is twice the sum of the first two angles. Find the measure of each angle.

\[(x) + (x + 10) + (4x + 20) = 180\]

\[6x + 30 = 180\]

\[6x = 150\]

\[x = 25\]

The angles are 25°, 35°, and 120°.

29. One of two complementary angles measures 30° more than three times the other. Find the measure of each angle.

\[(x) + (3x + 30) = 90\]

\[4x + 30 = 90\]

\[4x = 60\]

\[x = 15\]

The angles are 15° and 75°.

30. Find the measure of angle that is 10° more than its complement.

\[(x) + (x + 10) = 90\]

\[2x + 10 = 90\]

\[2x = 80\]

\[x = 40\]

The angle is 50°.
31. Find the measure of an angle that is $30^\circ$ less than its supplement.

\[
(x) + (x - 30) = 180
\]

\[
2x - 30 = 180
\]

\[
2x = 210
\]

\[
x = 105
\]

The angle is $75^\circ$.

32. Find the measure of an angle that is one-half the measure of its complement.

\[
(x) + \left(\frac{1}{2} x\right) = 90
\]

\[
1.5x = 90
\]

\[
x = 60
\]

The measure of the angle is $30^\circ$.

33. Find the measure of an angle that is one-half the measure of its supplement.

\[
(x) + \left(\frac{1}{2} x\right) = 180
\]

\[
1.5x = 180
\]

\[
x = 120
\]

The measure of the angle is $60^\circ$. 
Angles and Algebra Checklist

1. On questions 1 through 8, did the student find the complement of each angle correctly?
   a. All eight (40 points)
   b. Seven of the eight (35 points)
   c. Six of the eight (30 points)
   d. Five of the eight (25 points)
   e. Four of the eight (20 points)
   f. Three of the eight (15 points)
   g. Two of the eight (10 points)
   h. One of the eight (5 points)

2. On questions 9 through 16, did the student find the supplement of each angle correctly?
   a. All eight (40 points)
   b. Seven of the eight (35 points)
   c. Six of the eight (30 points)
   d. Five of the eight (25 points)
   e. Four of the eight (20 points)
   f. Three of the eight (15 points)
   g. Two of the eight (10 points)
   h. One of the eight (5 points)

3. On questions 17 through 24, did the student find the third angle of each triangle correctly?
   a. All eight (40 points)
   b. Seven of the eight (35 points)
   c. Six of the eight (30 points)
   d. Five of the eight (25 points)
   e. Four of the eight (20 points)
   f. Three of the eight (15 points)
   g. Two of the eight (10 points)
   h. One of the eight (5 points)
4. On questions 25 through 33, did the student answer the questions correctly?

   a. All nine (45 points)
   b. Eight of the nine (40 points)
   c. Seven of the nine (35 points)
   d. Six of the nine (30 points)
   e. Five of the nine (25 points)
   f. Four of the nine (20 points)
   g. Three of the nine (15 points)
   h. Two of the nine (10 points)
   i. One of the nine (5 points)

Total Number of Points __________

A  149 points and above
B  132 points and above
C  115 points and above
D  99 points and above
F  98 points and below

Any score below C needs remediation!