Solving Right Triangles Using Trigonometry Examples

1. To solve a triangle means to find all the missing measures of the triangle. The trigonometric ratios can be used to solve a triangle. The ratio used depends upon what measures are given and what measures are missing. Sometimes, more than one ratio can be used.

2. Review the following trigonometric ratios with students.

\[
\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}
\]

\[
\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}
\]

\[
\tan \theta = \frac{\text{opposite}}{\text{adjacent}}
\]

3. **Thought Provoker** – Put the following questions on the board and have the students consider them during the lesson. After the examples are completed, have the students make comments on the questions.

   a. Question 1 – Can a right triangle be solved if the lengths of two sides are given? (YES)
   b. Question 2 – Can a right triangle be solved if the measures of both acute angles are given? (NO)
   c. Question 3 – Can a right triangle be solved if the length of one side and the measure of one acute angle are given? (YES)

4. **Example** – Find the measures of \( \angle A \) and \( \angle C \) in \( \triangle ABC \).

Because the lengths of all sides are given, the sine, cosine, or tangent ratio can be used. Suppose you use the sine ratio.

\[
\sin A = \frac{4}{5} \Rightarrow 0.8000
\]

Using the table or calculator, we see that the measure of \( \angle A \) is approximately 53°. Thus, the measure of \( \angle C \) is about (90° – 53°), or 37°.
5. Emphasize to students that they can check their answers on problems similar to example 4. The sum of the three angles in a triangle equal 180 degrees. If not, the student should go back and check work.

6. Ask the students the following question. In example 4, how would you use the cosine ratio to find the measure of $\angle A$?

\[
\cos A = \frac{3}{5} \\
\cos A = 0.6000 \\
\text{Then } \angle A \approx 53^\circ.
\]

7. Ask the students the following question. In example 4, how would you use the tangent ratio to find the measure of $\angle A$?

\[
\tan A = \frac{4}{3} \\
\tan A = 1.3333 \\
\text{Then } \angle A \approx 53^\circ
\]

Point out that when the measures of all three sides of the triangle are known, any of the three trigonometric ratios can be used to find the measures of the acute angles.

8. Example – Find the measures of $\angle A$ and $\angle C$ in $\triangle ABC$.

Answers may vary but one type of solution is:

\[
\sin \angle A = \frac{12}{13} \\
\sin \angle A = 0.9231 \\
\text{Then } \angle A \approx 67^\circ
\]

\[
\sin \angle C = \frac{5}{13} \\
\sin \angle C = 0.3846 \\
\text{Then } \angle C \approx 23^\circ
\]

9. Example – Find the length of $DE$ in $\triangle DEF$.

With respect to $\angle E$, only the lengths of the adjacent side and the hypotenuse are given. In this case, use the cosine ratio.

\[
\cos 40^\circ = \frac{x}{7.6} \Rightarrow 0.7660 \approx \frac{x}{7.6} \Rightarrow (0.7660)(7.6) \approx x
\]

Thus, $DE$ is approximately 5.8 cm long.
10. Have students use \( \sin 50^\circ = \frac{x}{7.6} \) to solve for the measure of \( \overline{DE} \). (See example 9 above.) Do you get the same result? **YES**

11. **EXAMPLE** – Find the length of \( \overline{JK} \) in \( \triangle JKL \).

   In this case, use the tangent ratio.
   \[
   \tan 48^\circ = \frac{5.8}{x}
   \]
   \[
   1.1106 \approx \frac{5.8}{x}
   \]
   \[
   1.1106x \approx 5.8
   \]
   \[
   x \approx 5.2
   \]
   Thus, \( \overline{JK} \) is about 5.2 mm long.

12. Ask the students to explain why the tangent ratio is used in example 11.

   **The tangent ratio is used because the problem involves the measures of the two legs of the triangle.**

13. **Example** – Find the missing measures in the triangle at the right.

   The measure of \( \angle R \) is \( (90^\circ - 36^\circ) \), or \( 54^\circ \).
   \[
   \sin 36^\circ = \frac{x}{18}
   \]
   \[
   0.5878 \approx \frac{x}{18}
   \]
   \[
   10.6 \approx x
   \]
   Thus, \( \overline{QR} \) is about 10.6 inches long.

   \[
   \cos 36^\circ = \frac{y}{18}
   \]
   \[
   0.8090 \approx \frac{y}{18}
   \]
   \[
   14.56 \approx y
   \]
   Thus, \( \overline{PQ} \) is about 14.56 inches long.

Point out that once the measures of all sides are known, the Pythagorean Theorem can be used to check them.
14. **Example** – Find DE in $\triangle DEF$.

\[
\cos 36^\circ = \frac{x}{5.2}
\]

\[
0.8090 = \frac{x}{5.2}
\]

\[
x \approx 4.2
\]

Thus, $DE$ is about 4.2 m.

15. **Example** – Find JK in $\triangle JKL$.

\[
\tan 33^\circ = \frac{13.1}{x}
\]

\[
0.6494 = \frac{13.1}{x}
\]

\[
0.6494x \approx 13.1
\]

\[
x \approx 20.2
\]

Thus, $JK$ is about 20.2 mm.

16. **Example** – Find the missing measures of $\triangle ABC$.

\[
\sin 50^\circ = \frac{y}{8.3}
\]

\[
0.7660 \approx \frac{y}{8.3}
\]

\[
8.3(0.7660) \approx y
\]

\[
y \approx 6.4
\]

\[
\cos 50^\circ = \frac{x}{8.3}
\]

\[
0.6428 \approx \frac{x}{8.3}
\]

\[
8.3(0.6428) \approx x
\]

\[
x \approx 5.3
\]

$\angle C = (90 - 50)^\circ$

$\angle C = 40^\circ$

Therefore, $AB \approx 5.3$, $CB \approx 6.4$, $\angle C = 40^\circ$
Solving Right Triangles Using Trigonometry Activity Sheet

State which trigonometric ratios you would use to find the missing measures in each triangle.

1. \(\triangle ABC\) with \(\angle B = 46^\circ\), \(a = 15\) ft, \(b\) ft, \(c\) ft. Find \(a\) and \(b\).

2. \(\triangle ABC\) with \(\angle C = 39^\circ\), \(y\) m, \(x\) m. Find \(x\) and \(y\).

3. \(\triangle ABC\) with \(\angle A = 13\) ft, \(\angle B = 5\) ft, \(c\) ft. Find \(A\) and \(B\).

4. \(\triangle ABC\) with \(\angle C = 12\) in., \(\angle B = 15\) ft, \(\angle A = 9\) in. Find \(A\) and \(B\).

Solve Each Triangle. (When using trigonometric functions, approximate values to 4 decimal places and approximate degree measure to nearest whole degree).

5. \(\triangle ABC\) with \(\angle A = 21^\circ\), \(\angle C = 13\) in., \(\angle B = 18\) in. Find \(\angle A\) and \(\angle B\).

6. \(\triangle ABC\) with \(\angle A = 8\) ft, \(\angle B = 6\) ft, \(\angle C = 12\) ft. Find \(\angle A\) and \(\angle B\).

7. \(\triangle ABC\) with \(\angle A = 70^\circ\), \(\angle C = 9\) cm, \(\angle B = 18\) cm. Find \(\angle A\) and \(\angle B\).

8. \(\triangle ABC\) with \(\angle A = 60^\circ\), \(\angle B = 16\) m, \(\angle C = 30\) m. Find \(\angle A\) and \(\angle B\).
Draw and label, and then solve each right triangle ($\angle C$ is a right angle).

13. Angle $A = 31^\circ$, $a = 6\text{ m}$
14. $a = 6\text{ in.}$, $c = 10\text{ in.}$
15. Angle $B = 42^\circ$, $c = 10\text{ in.}$
16. $b = 5\text{ ft}$, $a = 4\text{ ft}$
17. $c = 14\text{ ft}$, $b = 11\text{ ft}$
18. $c = 11\text{ m}$, $b = 6\text{ m}$
19. Angle $B = 40^\circ$, $b = 6\text{ cm}$
20. Angle $B = 28^\circ$, $a = 16\text{ cm}$
21. Angle $A = 45^\circ$, $c = \sqrt{2}\text{ ft}$
22. Angle $A = 75^\circ$, $b = 3\text{ km}$
Solving Right Triangles Using Trigonometry
Activity Sheet Key

State which trigonometric ratios you would use to find the missing measures in each triangle.

1. 
\[
\begin{align*}
\text{Sin } 46^\circ &= \frac{b}{15} \\
\text{Cos } 46^\circ &= \frac{a}{15}
\end{align*}
\]

2. 
\[
\begin{align*}
\text{Cos } 39^\circ &= \frac{12}{y} \\
\text{Tan } 39^\circ &= \frac{x}{12}
\end{align*}
\]

3. 
\[
\begin{align*}
\text{Sin } A &= \frac{5}{13} \\
\text{Cos } B &= \frac{5}{13}
\end{align*}
\]

4. 
\[
\begin{align*}
\text{Tan } A &= \frac{12}{9} \\
\text{Tan } B &= \frac{9}{12}
\end{align*}
\]

Solve Each Triangle.

5. 
\[
\begin{align*}
\angle B &= (90 - 21)^\circ = 69^\circ \\
\text{Cos } 21^\circ &= \frac{13}{c} \approx 0.9336 \Rightarrow c \approx 13.9 \text{ in.} \\
\text{Tan } 21^\circ &= \frac{a}{13} \approx 0.3839 \Rightarrow a \approx 5 \text{ in.}
\end{align*}
\]

6. 
\[
\begin{align*}
\text{Tan } A &= \frac{8}{6} \Rightarrow \angle A \approx 53^\circ \\
\angle B \approx (90 - 53)^\circ \Rightarrow \angle B \approx 37^\circ \\
\text{Sin } 53^\circ &= \frac{8}{c} \Rightarrow 0.7986 \approx \frac{8}{c} \\
c \approx 10 \text{ ft}
\end{align*}
\]
7. \[ \angle B = (90 - 70)^\circ \Rightarrow 20^\circ \]
\[
\sin 70^\circ = \frac{9}{c} \Rightarrow 0.9397 \approx \frac{9}{c} \Rightarrow c \approx 9.6 \text{ cm}
\]
\[
\tan 70^\circ = \frac{9}{b} \Rightarrow 2.7475 \approx \frac{9}{b} \Rightarrow b \approx 3.3 \text{ cm}
\]

8. \[ \angle A = (90 - 60)^\circ \Rightarrow 30^\circ \]
\[
\sin 60^\circ = \frac{b}{16} \Rightarrow 0.8660 \approx \frac{b}{16} \Rightarrow b \approx 13.9 \text{ m}
\]
\[
\cos 60^\circ = \frac{a}{16} \Rightarrow 0.5000 \approx \frac{a}{16} \Rightarrow a \approx 8 \text{ m}
\]

9. \[ \angle B = (90 - 22)^\circ \Rightarrow 68^\circ \]
\[
\tan 22^\circ = \frac{a}{12} \Rightarrow 0.4040 \approx \frac{a}{12} \Rightarrow a \approx 4.8 \text{ ft}
\]
\[
\cos 22^\circ = \frac{12}{c} \Rightarrow 0.9272 \approx \frac{12}{c} \Rightarrow c \approx 12.9 \text{ ft}
\]

10. \[ \angle A = (90 - 52)^\circ \Rightarrow 38^\circ \]
\[
\sin 52^\circ = \frac{b}{23} \Rightarrow 0.7880 \approx \frac{b}{23} \Rightarrow b \approx 18.1 \text{ cm}
\]
\[
\cos 52^\circ = \frac{a}{23} \Rightarrow 0.6157 \approx \frac{a}{23} \Rightarrow a \approx 14.2 \text{ cm}
\]
11. \( \triangle ABC \) with \( \angle C = 90^\circ \):

\[
\angle B = (90 - 40)^\circ \rightarrow 50^\circ
\]

\[
\sin 40^\circ = \frac{a}{16} \Rightarrow 0.6428 \approx \frac{a}{16} \Rightarrow a \approx 10.3 \text{ m}
\]

\[
\cos 40^\circ = \frac{b}{16} \Rightarrow 0.7660 \approx \frac{b}{16} \Rightarrow b \approx 12.3 \text{ m}
\]

12. \( \triangle ABC \) with \( \angle C = 90^\circ \):

\[
\angle B = (90 - 35)^\circ \rightarrow 55^\circ
\]

\[
\sin 35^\circ = \frac{7}{c} \Rightarrow 0.5736 \approx \frac{7}{c} \Rightarrow c \approx 12.2 \text{ km}
\]

\[
\tan 35^\circ = \frac{7}{b} \Rightarrow 0.7002 \approx \frac{7}{b} \Rightarrow b \approx 10 \text{ km}
\]

Draw and label, and then solve each right triangle (\( \angle C \) is a right angle).

(Students’ drawings will vary.)

13. Angle \( A = 31^\circ \), \( a = 6 \text{ m} \):

\[
\angle B = (90 - 31)^\circ \rightarrow 59^\circ
\]

\[
\tan 31^\circ = \frac{6}{b} \Rightarrow 0.6009 \approx \frac{6}{b} \Rightarrow b \approx 10 \text{ m}
\]

\[
\sin 31^\circ = \frac{6}{c} \Rightarrow 0.5150 \approx \frac{6}{c} \Rightarrow c \approx 11.7 \text{ m}
\]

14. \( a = 6 \text{ in.} \), \( c = 10 \text{ in.} \):

\[
\sin A = \frac{6}{10} \Rightarrow \sin A = 0.6000 \Rightarrow A \approx 37^\circ
\]

\[
\angle B \approx (90 - 37)^\circ \rightarrow 53^\circ
\]

\[
\tan 37^\circ \approx \frac{6}{b} \Rightarrow 0.7536 \approx \frac{6}{b} \Rightarrow b \approx 8 \text{ in.}
\]
15. Angle B = 42°, c = 10 in.

\[ \angle A = (90 - 42)^\circ \Rightarrow 48^\circ \]
\[ \sin 42^\circ = \frac{b}{10} \Rightarrow 0.6691 \approx \frac{b}{10} \Rightarrow b \approx 6.7 \text{ in.} \]
\[ \cos 42^\circ = \frac{a}{10} \Rightarrow 0.7431 \approx \frac{a}{10} \Rightarrow a \approx 7.4 \text{ in.} \]

16. b = 5 ft, a = 4 ft

\[ \tan A = \frac{4}{5} \Rightarrow \tan A \approx 0.8000 \Rightarrow \angle A \approx 39^\circ \]
\[ \angle B \approx (90 - 39)^\circ \Rightarrow \angle B \approx 51^\circ \]
\[ \sin 39^\circ \approx \frac{4}{c} \Rightarrow 0.6293 \approx \frac{4}{c} \Rightarrow c \approx 6.4 \text{ ft} \]

17. c = 14 ft, b = 11 ft

\[ \cos A = \frac{11}{14} \Rightarrow \cos A \approx 0.7857 \Rightarrow \angle A \approx 38^\circ \]
\[ \angle B \approx (90 - 38)^\circ \Rightarrow \angle B \approx 52^\circ \]
\[ \sin 38^\circ \approx \frac{a}{14} \Rightarrow 0.6157 \approx \frac{a}{14} \Rightarrow a \approx 8.6 \text{ ft} \]

18. c = 11 m, b = 6 m

\[ \cos A = \frac{6}{11} \Rightarrow \cos A \approx 0.5454 \Rightarrow \angle A \approx 57^\circ \]
\[ \angle B \approx (90 - 57)^\circ \Rightarrow \angle B \approx 33^\circ \]
\[ \sin 57^\circ \approx \frac{a}{11} \Rightarrow 0.8387 \approx \frac{a}{11} \Rightarrow a \approx 9.2 \text{ m} \]

19. Angle B = 40°, b = 6 cm

\[ \angle A = (90 - 40)^\circ \Rightarrow \angle A = 50^\circ \]
\[ \sin 40^\circ = \frac{6}{c} \Rightarrow 0.6428 \approx \frac{6}{c} \Rightarrow c \approx 9.3 \text{ cm} \]
\[ \tan 40^\circ = \frac{6}{a} \Rightarrow 0.8391 \approx \frac{6}{a} \Rightarrow a \approx 7.2 \text{ cm} \]
20. Angle B = 28°, a = 16 cm

\[ \angle A = (90 - 28)^\circ \Rightarrow \angle A = 62^\circ \]

\[
\tan 28^\circ = \frac{b}{16} \approx 0.5317 \Rightarrow b \approx 8.5 \text{ cm}
\]

\[
\cos 28^\circ = \frac{16}{c} \approx 0.8829 \Rightarrow c \approx 18.1 \text{ cm}
\]

21. Angle A = 45°, c = \( \sqrt{2} \) ft

\[ \angle B = (90 - 45)^\circ \Rightarrow \angle B = 45^\circ \]

\[
\sin 45^\circ = \frac{a}{\sqrt{2}} \approx 0.7071 \Rightarrow a \approx 1 \text{ ft}
\]

\[
\cos 45^\circ = \frac{b}{\sqrt{2}} \approx 0.7071 \Rightarrow b \approx 1 \text{ ft}
\]

22. Angle A = 75°, b = 3 km

\[ \angle B = (90 - 75)^\circ \Rightarrow \angle B = 15^\circ \]

\[
\tan 75^\circ = \frac{a}{3} \approx 3.7321 \Rightarrow a \approx 11.2 \text{ km}
\]

\[
\cos 75^\circ = \frac{3}{c} \approx 0.2588 \Rightarrow c \approx 11.6 \text{ km}
\]
Solving Right Triangles Using Trigonometry Checklist

1. On questions 1, did the student use correct ratios to find missing measures?
   a. Both (10 points)
   b. One of the two (5 points)

2. On questions 2, did the student use correct ratios to find missing measures?
   a. Both (10 points)
   b. One of the two (5 points)

3. On questions 3, did the student use correct ratios to find missing measures?
   a. Both (10 points)
   b. One of the two (5 points)

4. On questions 4, did the student use correct ratios to find missing measures?
   a. Both (10 points)
   b. One of the two (5 points)

5. On question 5, did the student solve the triangle for the missing measures correctly? (3 missing parts)
   a. All three (15 points)
   b. Two of the three (10 points)
   c. One of the three (5 points)

6. On question 6, did the student solve the triangle for the missing measures correctly? (3 missing parts)
   a. All three (15 points)
   b. Two of the three (10 points)
   c. One of the three (5 points)

7. On question 7, did the student solve the triangle for the missing measures correctly? (3 missing parts)
   a. All three (15 points)
   b. Two of the three (10 points)
   c. One of the three (5 points)
8. On question 8, did the student solve the triangle for the missing measures correctly? (3 missing parts)
   a. All three (15 points)
   b. Two of the three (10 points)
   c. One of the three (5 points)

9. On question 9, did the student solve the triangle for the missing measures correctly? (3 missing parts)
   a. All three (15 points)
   b. Two of the three (10 points)
   c. One of the three (5 points)

10. On question 10, did the student solve the triangle for the missing measures correctly? (3 missing parts)
    a. All three (15 points)
    b. Two of the three (10 points)
    c. One of the three (5 points)

11. On question 11, did the student solve the triangle for the missing measures correctly? (3 missing parts)
    a. All three (15 points)
    b. Two of the three (10 points)
    c. One of the three (5 points)

12. On question 12, did the student solve the triangle for the missing measures correctly? (3 missing parts)
    a. All three (15 points)
    b. Two of the three (10 points)
    c. One of the three (5 points)

13. On question 13, did the student draw, label, and solve the triangle for the missing measures correctly? (4 parts – drawing and finding 3 measures).
    a. All four (20 points)
    b. Three of the four (15 points)
    c. Two of the four (10 points)
    d. One of the four (5 points)
14. On question 13, did the student draw, label, and solve the triangle for the missing measures correctly? (4 parts – drawing and finding 3 measures).
   
   a. All four (20 points)  
   b. Three of the four (15 points)  
   c. Two of the four (10 points)  
   d. One of the four (5 points)

15. On question 15, did the student draw, label, and solve the triangle for the missing measures correctly? (4 parts – drawing and finding 3 measures).
   
   a. All four (20 points)  
   b. Three of the four (15 points)  
   c. Two of the four (10 points)  
   d. One of the four (5 points)

16. On question 16, did the student draw, label, and solve the triangle for the missing measures correctly? (4 parts – drawing and finding 3 measures).
   
   a. All four (20 points)  
   b. Three of the four (15 points)  
   c. Two of the four (10 points)  
   d. One of the four (5 points)

17. On question 17, did the student draw, label, and solve the triangle for the missing measures correctly? (4 parts – drawing and finding 3 measures).
   
   a. All four (20 points)  
   b. Three of the four (15 points)  
   c. Two of the four (10 points)  
   d. One of the four (5 points)

18. On question 18, did the student draw, label, and solve the triangle for the missing measures correctly? (4 parts – drawing and finding 3 measures).
   
   a. All four (20 points)  
   b. Three of the four (15 points)  
   c. Two of the four (10 points)  
   d. One of the four (5 points)
19. On question 19, did the student draw, label, and solve the triangle for the missing measures correctly? (4 parts – drawing and finding 3 measures).
   a. All four (20 points)
   b. Three of the four (15 points)
   c. Two of the four (10 points)
   d. One of the four (5 points)

20. On question 20, did the student draw, label, and solve the triangle for the missing measures correctly? (4 parts – drawing and finding 3 measures).
   a. All four (20 points)
   b. Three of the four (15 points)
   c. Two of the four (10 points)
   d. One of the four (5 points)

21. On question 21, did the student draw, label, and solve the triangle for the missing measures correctly? (4 parts – drawing and finding 3 measures).
   a. All four (20 points)
   b. Three of the four (15 points)
   c. Two of the four (10 points)
   d. One of the four (5 points)

22. On question 22, did the student draw, label, and solve the triangle for the missing measures correctly? (4 parts – drawing and finding 3 measures).
   a. All four (20 points)
   b. Three of the four (15 points)
   c. Two of the four (10 points)
   d. One of the four (5 points)

Total Number of Points __________

A  252 points and above
B  224 points and above
C  196 points and above
D  168 points and above
F  167 points and below

Any score below C needs remediation!