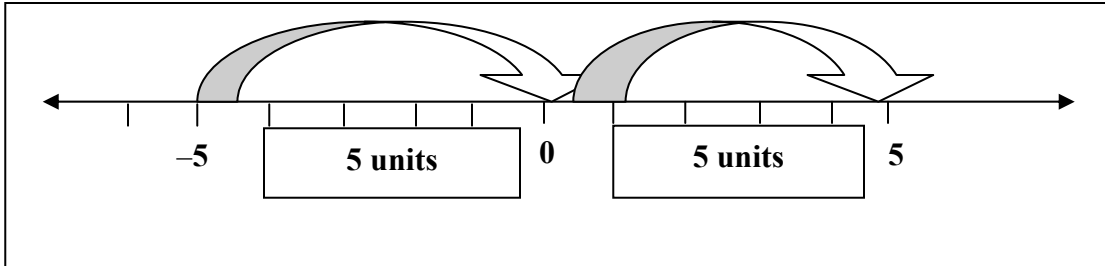
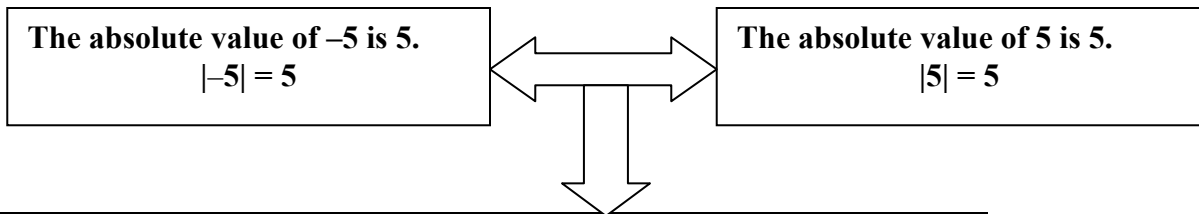


Solving Absolute Value Equations Examples

1. Even though the numbers -5 and 5 are different, they do have something in common. They are the same distance from 0 on the number line, but in opposite directions.



2. We say that -5 and 5 have the same absolute value. The absolute value of a number is the number of units it is from 0 on the number line.



Definition of Absolute Value	<p>For any real number a:</p> <p>If $a \geq 0$, then $a = a$</p> <p>If $a < 0$, then $a = -a$</p>
-------------------------------------	--

3. **Thought Provoker** – What is the value of $\frac{x}{|x|}$ when x is positive? → 1

4. **Thought Provoker** – What is the value of $\frac{x}{|x|}$ when x is negative? → -1

5. **Example** – Find the absolute value of 3 and -7 .

$ 3 = 3$ $ -7 = 7$

6. **Example** – Find the absolute value of $x - 5$.

Using the definition of absolute value:

If $(x - 5)$ is greater than or equal to zero, then $|x - 5| = x - 5$

If $(x - 5)$ is less than zero, then $|x - 5| = -(x - 5)$ or $-x + 5$

7. **Example** – Evaluate $|3x - 2| + 7.2$ if $x = -3$.

$$\begin{aligned} &|3x - 2| + 7.2 \\ &|-9 - 2| + 7.2 \\ &|-11| + 7.2 \\ &11 + 7.2 \\ &18.2 \end{aligned}$$

8. **Example** – Find the absolute value of $3 - 7$.

$$\begin{aligned} &|3 - 7| \\ &|-4| \\ &4 \end{aligned}$$

9. **Example** – Find the absolute value of $2x$.

$$\begin{aligned} &\text{If } 2x \geq 0 \text{ then } |2x| = 2x \\ &\text{If } 2x < 0 \text{ then } |2x| = -2x \end{aligned}$$

10. **Example** – Evaluate $|x + 4|$ if $x = -6$

$$\begin{aligned} &|x + 4| \\ &|-6 + 4| \\ &|-2| \\ &2 \end{aligned}$$

11. **Example** – Evaluate $|-8a - 3|$ if $a = -2$

$$\begin{aligned} &|-8a - 3| \\ &|-8(-2) - 3| \\ &|16 - 3| \\ &|13| \\ &13 \end{aligned}$$

12. **Example** – Evaluate $|y - 3| - |-2y|$ if $y = 4$

$$\begin{aligned} &|y - 3| - |-2y| \\ &|4 - 3| - |-2(4)| \\ &|1| - |-8| \\ &1 - 8 \\ &-7 \end{aligned}$$

13. **Example** – Evaluate $-3|2t + 6|$ if $t = -1$

$$\begin{aligned} &-3|2t + 6| \\ &-3|2(-1) + 6| \\ &-3|-2 + 6| \\ &-3|4| \\ &-3(4) \\ &-12 \end{aligned}$$

14. **Example** – Solve $|x - 7| = 12$. Check each solution.

If $x - 7 \geq 0$ then $x - 7 = 12$

$$x = 19$$

Check – Is $|19 - 7| = 12$ YES

If $x - 7 < 0$ then $x - 7 = -12$

$$x = -5$$

Check – Is $|-5 - 7| = 12$ YES

The solution set is $\{19, -5\}$

Point out that since $|12| = 12$ and $|-12| = 12$ then $|x - 7| = 12$ intuitively means that $x - 7 = 12$ and $x - 7 = -12$. The equation $|x - 7| = 12$ can also be solved using $x - 7 = 12$ or $-(x - 7) = 12$.

15. **Example** – Solve $5|2x + 3| = 30$. Check each solution.

Rewrite by dividing both sides by 5
 $|2x + 3| = 6$

If $2x + 3 \geq 0$ then $2x + 3 = 6$

$$2x = 3 \rightarrow x = \frac{3}{2}$$

Check – Is $|2(\frac{3}{2}) + 3| = 6$ YES

If $2x + 3 < 0$ then $2x + 3 = -6$

$$2x = -9 \rightarrow x = -\frac{9}{2}$$

Check – Is $|2(-\frac{9}{2}) + 3| = 6$ YES

The solution set is $\{\frac{3}{2}, -\frac{9}{2}\}$

16. **EXAMPLE** – Solve $|k + 6| = 9$. Check each solution.

If $k + 6 \geq 0$ then $k + 6 = 9$

$$k = 3$$

Check – Is $|3 + 6| = 9$ YES

If $k + 6 < 0$ then $k + 6 = -9$

$$k = -15$$

**Check – Is $|-15 + 6| = 9$
 $|-9| = 9$ YES**

**The solution set is
 $\{3, -15\}$**

17. **Example** – Solve $-2|x + 2| + 12 = 0$. Check each solution.

**Rewrite equation by subtracting 12 from
both sides and dividing by negative 2.**

$$-2|x + 2| = -12 \rightarrow |x + 2| = 6$$

If $x + 2 \geq 0$ then $x + 2 = 6$

$$x = 4$$

Check – Is $-2|4 + 2| + 12 = 0$

If $x + 2 < 0$ then $x + 2 = -6$

$$x = -8$$

Check – Is $|-8 + 2| = 6$ YES

**The solution set is
 $\{4, -8\}$**

18. **Example** – Solve $-2|m - 3| + 8 = -24$

**Rewrite equation by subtracting eight from
both sides and dividing by negative two.**

$$-2|m - 3| = -32 \rightarrow |m - 3| = 16$$

If $m - 3 \geq 0$ then $m - 3 = 16$

$$m = 19$$

Check – Is $|19 - 3| = 16$ YES

If $m - 3 < 0$ then $m - 3 = -16$

$$m = -13$$

Check – Is $|-13 - 3| = 16$ YES

**The solution set is
 $\{19, -13\}$**

19. An **absolute value** equation may have **no solution**. For example, $|x| = -3$ is never true. Since the absolute value of a number is always positive or zero, there is **no replacement** for x that will make the sentence true. The equation $|x| = -3$ has no solution. The solution set has no members at all. This solution set is called the **empty set** and is symbolized by either $\{ \}$ or \emptyset .

Another name for \emptyset is the null set.

20. **Example** – Solve $|3x + 7| + 4 = 0$. Check each solution.

First rewrite the equation by subtracting 4 from each side.

$$|3x + 7| = -4$$

This sentence is never true, so the equation has no solution. The solution set is \emptyset .

21. **Example** – Solve $-2|x + 3| = 6$. Check each solution.

First, rewrite the equation by dividing each side by negative 2.

$$|x + 3| = -3.$$

This sentence is never true, so the equation has no solution. The solution set is \emptyset .

22. It is important to check your answers when solving absolute value equations. Even if the correct procedure for solving the equation is used, the answers may not be actual solutions to the original equation.

Be especially careful when equations have variables on both sides.

23. **Example** – Solve $|2x + 12| = 7x - 3$. Check each solution.

If $2x + 12 \geq 0$ then $2x + 12 = 7x - 3$
 $15 = 5x$
 $3 = x$

Check – Is $|2(3) + 12| = 7(3) - 3$
 $|6 + 12| = 18$ YES

If $2x + 12 < 0$ then $2x + 12 = -(7x - 3)$
 $2x + 12 = -7x + 3 \rightarrow 9x = -9$
 $x = -1$

Check – Is $|2(-1) + 12| = 7(-1) - 3$
 $|10| = -10$ NO

Note that $7x - 3$ must be nonnegative. Thus, $x \geq \frac{3}{7}$. Since -1 is not permissible, the only solution is 3.

The solution set is $\{3\}$

24. **Example** – Solve $|x + 6| = 2x$. Check each solution.

If $x + 6 \geq 0$ then $x + 6 = 2x$
 $6 = x$

Check – Is $|6 + 6| = 2(6)$
 $12 = 12$ YES

If $x + 6 < 0$ then $x + 6 = -(2x)$
 $-3x = 6 \rightarrow x = -2$

Check – Is $|-2 + 6| = 2(-2)$
 $4 = -4$ NO

The solution set is $\{6\}$.

25. Using a calculator to perform absolute value computations.

Evaluate $7|6y - 8|$ when $y = 1$

First evaluate $6y - 8$.

ENTER: 6 1 8

DISPLAY: 6 1 6 8 -2

Since an absolute value cannot be negative, change the sign of the number in the display. Then, multiply this number by 7.

ENTER: 7

DISPLAY: -2 2 7 14

The value is 14.

Mention to students that some calculators have an absolute value key, This allows them to quickly find the absolute value of a number shown in the display. Be sure students understand when the change-sign key can be used. Students may make the mistake of changing the sign when they see the first negative number in the display. They must use the key only after the expression in the absolute value is completely evaluated. For example, when evaluating $7|6(-2) - 8|$ the students might try to use the change-sign key after obtaining -12 as the product of 6 and -2 . Emphasize that students must subtract 8 from -12 before using the change-sign key.

Name: _____

Date: _____

Class: _____

Solving Absolute Value Equations Worksheet

Evaluate if $x = -5$

1. $|x|$

2. $|4x|$

3. $|-2x|$

4. $|x + 6|$

5. $|7x - 1|$

6. $|-x|$

7. $|2x + 5|$

8. $|-2x + 5|$

9. $5 - |x|$

10. $5 - |-x|$

11. $|x| + x$

12. $|x - 7| - 8$

13. $7 - |3x + 10|$

14. $|x + 4| + |2x|$

Solve each equation. Check Solutions.

15. $|x + 11| = 42$

16. $|x - 5| = 11$

17. $3|x + 7| = 36$

18. $8|x - 3| = 88$

19. $|\frac{1}{2}x + 2| = 8$

20. $|x - \frac{7}{3}| = 6$

21. $\frac{1}{3}|6x + 5| = 7$

22. $|2x + 9| = 30$

23. $|4x - 3| = -27$

24. $|2x + 7| = 0$

25. $-6|2x - 14| = -42$

26. $|2a + 7| = a - 4$

27. $|7 + 3a| = 11 - a$

Use a calculator to evaluate each expression.

28. $|7(-3) + 10|$

29. $|7(-3)| + 10$

30. $3|4x - 9|$ if $x = 2.8$

31. $3|4x - 9|$ if $x = 2.2$

32. $48|7k - 30|$ if $k = 14$

33. $4 - |5n - 8|$ if $n = 2$

Solving Absolute Value Equations Worksheet Key

Evaluate if $x = -5$

- | | |
|------------------------------|-------------------------------------|
| 1. $ x \rightarrow 5$ | 8. $ -2x + 5 \rightarrow 15$ |
| 2. $ 4x \rightarrow 20$ | 9. $5 - x \rightarrow 0$ |
| 3. $ -2x \rightarrow 10$ | 10. $5 - -x \rightarrow 0$ |
| 4. $ x + 6 \rightarrow 1$ | 11. $ x + x \rightarrow 0$ |
| 5. $ 7x - 1 \rightarrow 36$ | 12. $ x - 7 - 8 \rightarrow 4$ |
| 6. $ -x \rightarrow 5$ | 13. $7 - 3x + 10 \rightarrow 2$ |
| 7. $ 2x + 5 \rightarrow 5$ | 14. $ x + 4 + 2x \rightarrow 11$ |

Solve each equation. Check Solutions.

15. $|x + 11| = 42$

If $x + 11 \geq 0$ then $x + 11 = 42$

$x = 31$

Check – Is $|31 + 11| = 42$ YES

If $x + 11 < 0$ then $x + 11 = -42$

$x = -53$

Check – Is $|31 + (-53)| = 42$ YES

**The solution set is
 $\{31, -53\}$**

16. $|x - 5| = 11$

If $x - 5 \geq 0$ then $x - 5 = 11$

$x = 16$

Check – Is $|16 - 5| = 11$ YES

If $x - 5 < 0$ then $x - 5 = -11$

$x = -6$

Check – Is $|-6 - 5| = 11$ YES

**The solution set is
 $\{16, -6\}$**

17. $3|x + 7| = 36$

Rewrite equation by dividing each side by 3.
 $|x + 7| = 12$

If $x + 7 \geq 0$ then $x + 7 = 12$

$x = 5$

Check – Is $3|5 + 7| = 36$ YES

If $x + 7 < 0$ then $x + 7 = -12$

$x = -19$

Check – Is $3|-19 + 7| = 36$ YES

**The solution set is
 $\{5, -19\}$**

18. $8|x - 3| = 88$

Rewrite equation by dividing each side by 8.

$|x - 3| = 11$

If $x - 3 \geq 0$ then $x - 3 = 11$

$x = 14$

Check - Is $8|14 - 3| = 88$ YES

If $x - 3 < 0$ then $x - 3 = -11$

$x = -8$

Check - Is $8|-8 - 3| = 88$ YES

Solution set is
 $\{14, -8\}$

19. $|\frac{1}{2}x + 2| = 8$

If $\frac{1}{2}x + 2 \geq 0$ then $\frac{1}{2}x + 2 = 8$

$\frac{1}{2}x = 6 \rightarrow x = 12$

Check - Is $|\frac{1}{2}(12) + 2| = 8$ YES

If $\frac{1}{2}x + 2 < 0$ then $\frac{1}{2}x + 2 = -8$

$\frac{1}{2}x = -10 \rightarrow x = -20$

Check - Is $|\frac{1}{2}(-20) + 2| = 8$ YES

Solution set is
 $\{12, -20\}$

20. $|x - \frac{7}{3}| = 6$

If $x - \frac{7}{3} \geq 0$ then $x - \frac{7}{3} = 6$

$x = \frac{25}{3}$

Check - Is $|\frac{25}{3} - \frac{7}{3}| = 6$ YES

If $x - \frac{7}{3} < 0$ then $x - \frac{7}{3} = -6$

$x = -\frac{11}{3}$

Check - Is $|\frac{11}{3} - \frac{7}{3}| = 6$ YES

Solution set is
 $\{\frac{25}{3}, -\frac{11}{3}\}$

21. $\frac{1}{3}|6x + 5| = 7$

Rewrite the equation by multiplying each side by 3.

$|6x + 5| = 21$

If $6x + 5 \geq 0$ then $6x + 5 = 21$

$6x = 16 \rightarrow x = \frac{8}{3}$

Check - Is $\frac{1}{3}|6(\frac{8}{3}) + 5| = 7$ YES

If $6x + 5 < 0$ then $6x + 5 = -21$

$6x = -26 \rightarrow x = -\frac{13}{3}$

Check - Is $\frac{1}{3}|6(-\frac{13}{3}) + 5| = 7$ YES

Solution set is
 $\{\frac{8}{3}, -\frac{13}{3}\}$

$$22. |2x + 9| = 30$$

If $2x + 9 \geq 0$ then $2x + 9 = 30$

$$2x = 21 \rightarrow x = \frac{21}{2}$$

Check – Is $|2(\frac{21}{2}) + 9| = 30$ YES

If $2x + 9 < 0$ then $2x + 9 = -30$

$$2x = -39 \rightarrow x = \frac{-39}{2}$$

Check – Is $|2(\frac{-39}{2}) + 9| = 30$ YES

Solution set is

$$\{\frac{21}{2}, \frac{-39}{2}\}$$

$$23. |4x - 3| = -27$$

Since the absolute value of a number cannot be negative, there is no solution

Solution set is \emptyset

$$24. |2x + 7| = 0$$

Since the absolute value is equal to zero, there can only be one solution

$$2x + 7 = 0$$

$$2x = -7 \rightarrow x = -\frac{7}{2}$$

Check – Is $|2(-\frac{7}{2}) + 7| = 0$ YES

Solution set is $\{-\frac{7}{2}\}$

$$25. -6|2x - 14| = -42$$

Rewrite equation by dividing both sides by negative 6

$$|2x - 14| = 7$$

If $2x - 14 \geq 0$ then $2x - 14 = 7$

$$2x = 21 \rightarrow x = \frac{21}{2}$$

Check – Is $-6|2(\frac{21}{2}) - 14| = -42$ YES

If $2x - 14 < 0$ then $2x - 14 = -7$

$$2x = 7 \rightarrow x = \frac{7}{2}$$

Check – Is $-6|2(\frac{7}{2}) - 14| = -42$ YES

Solution set is $\{\frac{21}{2}, \frac{7}{2}\}$

$$26. |2a + 7| = a - 4$$

If $2a + 7 \geq 0$ then $2a + 7 = a - 4$

$$a = -11$$

Does not check because $(a - 4)$ must be positive

If $2a + 7 < 0$ then $2a + 7 = -(a - 4)$

$$3a = -3 \rightarrow a = -1$$

Does not check because $(a - 4)$ must be positive

$$27. |7 + 3a| = 11 - a$$

If $7 + 3a \geq 0$ then $7 + 3a = 11 - a$

$$4a = 4 \rightarrow a = 1$$

Check – Is $|7 + 3(1)| = 11 - (1)$ YES

If $7 + 3a < 0$ then $7 + 3a = -(11 - a)$

$$2a = -18 \rightarrow a = -9$$

Check – Is $|7 + 3(-9)| = 11 - (-9)$ YES

Use a calculator to evaluate each expression.

$$28. |7(-3) + 10| \rightarrow 11$$

$$29. |7(-3)| + 10 \rightarrow 31$$

$$30. 3|4x - 9| \text{ if } x = 2.8 \rightarrow 6.6$$

$$31. 3|4x - 9| \text{ if } x = 2.2 \rightarrow 0.6$$

$$32. 48|7k - 30| \text{ if } k = 14 \rightarrow 3264$$

$$33. 4 - |5n - 8| \text{ if } n = 2 \rightarrow 2$$

Student Name: _____

Date: _____

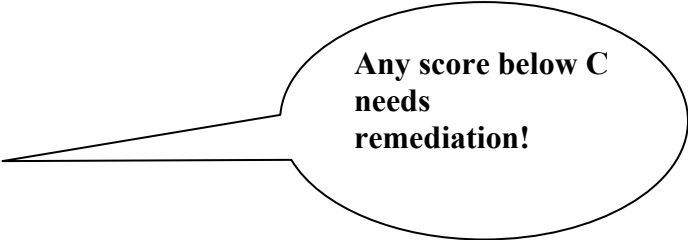
Solving Absolute Value Equations Checklist

1. On questions 1 thru 14, did the student evaluate the questions correctly?
 - a. All 14 (70 points)
 - b. 13 out of 14 (65 points)
 - c. 12 out of 14 (60 points)
 - d. 11 out of 14 (55 points)
 - e. 10 out of 14 (50 points)
 - f. 9 out of 14 (45 points)
 - g. 8 out of 14 (40 points)
 - h. 7 out of 14 (35 points)
 - i. 6 out of 14 (30 points)
 - j. 5 out of 14 (25 points)
 - k. 4 out of 14 (20 points)
 - l. 3 out of 14 (15 points)
 - m. 2 out of 14 (10 points)
 - n. 1 out of 14 (5 points)
2. On question 15, did the student solve the equation correctly and check solution?
 - a. Both (10 points)
 - b. Solved correctly but did not check solution (5 points)
3. On question 16, did the student solve the equation correctly and check solution?
 - a. Both (10 points)
 - b. Solved correctly but did not check solution (5 points)
4. On question 17, did the student solve the equation correctly and check solution?
 - a. Both (10 points)
 - b. Solved correctly but did not check solution (5 points)
5. On question 18, did the student solve the equation correctly and check solution?
 - a. Both (10 points)
 - b. Solved correctly but did not check solution (5 points)
6. On question 19, did the student solve the equation correctly and check solution?
 - a. Both (10 points)
 - b. Solved correctly but did not check solution (5 points)
7. On question 20, did the student solve the equation correctly and check solution?
 - a. Both (10 points)
 - b. Solved correctly but did not check solution (5 points)
8. On question 21, did the student solve the equation correctly and check solution?
 - a. Both (10 points)
 - b. Solved correctly but did not check solution (5 points)
9. On question 22, did the student solve the equation correctly and check solution?
 - a. Both (10 points)
 - b. Solved correctly but did not check solution (5 points)

10. On question 23, did the student solve the equation correctly and check solution?
- Both (10 points)
 - Solved correctly but did not check solution (5 points)
11. On question 24, did the student solve the equation correctly and check solution?
- Both (10 points)
 - Solved correctly but did not check solution (5 points)
12. On question 25, did the student solve the equation correctly and check solution?
- Both (10 points)
 - Solved correctly but did not check solution (5 points)
13. On question 26, did the student solve the equation correctly and check solution?
- Both (10 points)
 - Solved correctly but did not check solution (5 points)
14. On question 27, did the student solve the equation correctly and check solution?
- Both (10 points)
 - Solved correctly but did not check solution (5 points)
15. On questions 28 thru 33, did the student evaluate each problem correctly using a calculator?
- | | |
|---------------------------|---------------------------|
| a. All six (30 points) | d. 3 out of 6 (15 points) |
| b. 5 out of 6 (25 points) | e. 2 out of 6 (10 points) |
| c. 4 out of 6 (20 points) | f. 1 out of 6 (5 points) |

Total Number of Points _____

- A 252 points and above
- B 224 points and above
- C 196 points and above
- D 168 points and above
- F 167 points and below



NOTE: The sole purpose of this checklist is to aide the teacher in identifying students that need remediation. Students who meet the “C” criteria are ready for the next level of learning. The teacher, when assigning grades to the handout, may also use a percentage to assign grades.