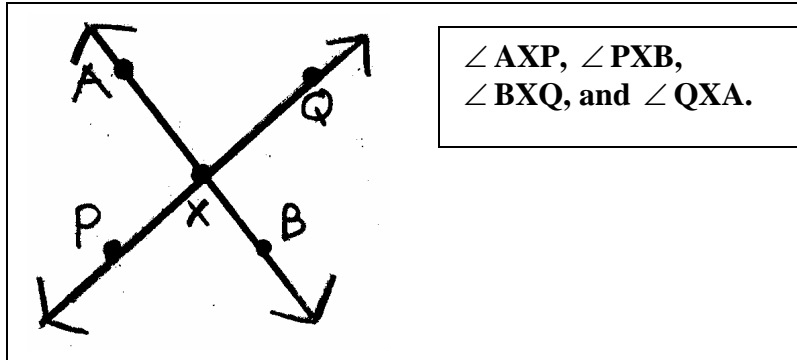


Pairs of Angles Examples

1. Introduce lesson by having students draw two lines \overline{AB} and \overline{PQ} , intersecting at point X. Have students look at the lines and name the four nonstraight angles that are formed. Indicate to students that these types of angles have special names and are used to describe relationships between the angles.



2. **Definition of Adjacent Angles** – $\angle AXP$ and $\angle PXB$ are **adjacent angles**.
Adjacent angles are angles in the same plane that have a common vertex and a common side, but no common interior points.

3. **Thought Provoker** – How many pairs of adjacent angles are in the figure above? → **4 pair**

$\angle AXP, \angle PXB;$
 $\angle PXB, \angle BXQ;$
 $\angle BXQ, \angle QXA;$
 $\angle QXA, \angle AXP$

4. **Definition of Vertical Angles** – $\angle AXP$ and $\angle BXQ$ are vertical angles. **Vertical angles** are two nonadjacent angles formed by two intersecting lines.

When defining vertical angles, point out that they have only one point in common – the vertex. Students may use this to help them remember what vertical angles are.

5. **Thought Provoker** – How many pairs of vertical angles are formed by the figure above? → **2 pair**

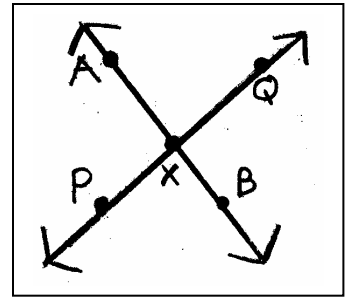
$\angle AXP, \angle BXA;$
 $\angle AXQ, \angle PXB$

6. Have students carefully measure the figure (see # 1 above) using a protractor. Then have students draw another pair of intersecting lines, measure the four angles, and compare results. Have students report their results.

Answers may vary but typical reports may include:

Vertical angles are congruent.
Adjacent angles are supplementary.
Adjacent angles form straight angles.

7. The original figure is again shown at the right. $\angle AXP$ and $\angle PXB$ are an example of a special pair of adjacent angles called a linear pair.
8. **Definition of a Linear Pair** – A **linear pair** of angles are adjacent angles whose noncommon sides are opposite rays.
9. **Ask the students the following**, “Based on the angle measurements you took previously of the above figure, what can you conclude about the measures of the angles in each linear pair?”



Sample response:

In this figure, there are four linear pairs: $\angle AXP$ and $\angle PXB$, $\angle PXB$ and $\angle BXQ$, $\angle BXQ$ and $\angle QXA$, $\angle QXA$ and $\angle AXP$.

10. Have students draw another pair of intersecting lines, measure the four angles, and see if they get the same results. Ask the students if these results lead them to any conclusions.

Help students reach the conclusion that:

The sum of the measures of a linear pair is 180° .

When discussing linear pairs of angles, point out that since linear means “deals with lines” and pair means “two,” another way to define linear pair would be “two angles that make a line.”

11. **Example** – In the figure at the right, \overline{AB} and \overline{CD} intersect at Z. Find the value of x and the measure of $\angle CZB$.

Since $\angle AZC$ and $\angle BZD$ are vertical angles,

$$m \angle AZC = m \angle BZD$$

$$5x - 22 = 3x + 16$$

$$2x = 38$$

$$x = 19$$

$$m \angle AZC = 5x - 22 \rightarrow 5(19) - 22$$

$$m \angle AZC = 73$$

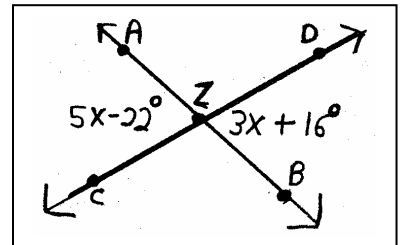
Since $\angle AZC$ and $\angle CZB$ are a linear pair,

$$m \angle AZC + m \angle CZB = 180$$

$$73 + m \angle CZB = 180$$

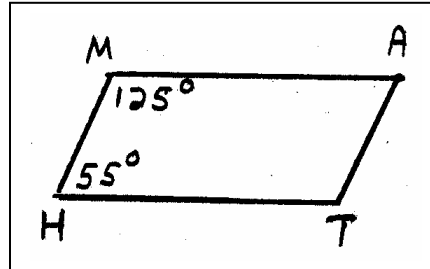
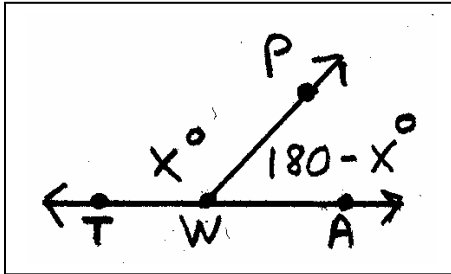
$$m \angle CZB = 107$$

The measure of $\angle CZB$ is 107°



12. **Definition of Supplementary Angles** – In the example 11, the sum of the measures of $\angle AZC$ and $\angle CZB$ is 180. If the sum of the measures of two angles is 180, the angles are called **supplementary angles**. When two angles are supplementary, each angle is said to be a **supplement** of the other angle.

13. Give students examples of supplementary angles.



Since the sum of the measures of a linear pair of angles is 180° , we can now say that two angles that form a linear pair must be supplementary

14. **Example** – If $\angle PQS$ and $\angle SQR$ are supplementary, find $m\angle SQT$ and $m\angle TQR$ (see diagram at right).

Since $\angle PQS$ and $\angle SQR$ are supplementary, the sum of their measures is 180.

$$m\angle PQS + m\angle SQR = 180$$

Substitute 90 for $m\angle PQS$

$$90 + m\angle SQR = 180$$

$$90 + (m\angle SQT + m\angle TQR) = 180$$

$$90 + (x + 28) + (6x - 15) = 180$$

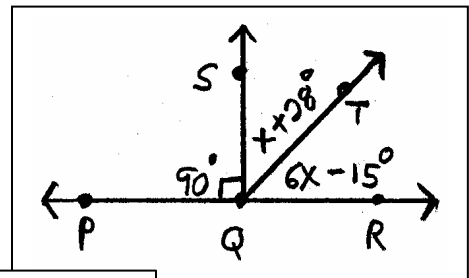
$$7x + 103 = 180$$

$$7x = 77$$

$$x = 11$$

$$m\angle SQT = x + 28 \rightarrow (11) + 28 \rightarrow 39$$

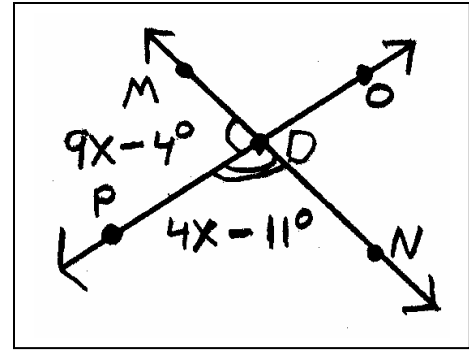
$$m\angle TQR = 6x - 15 \rightarrow 6(11) - 15 \rightarrow 51$$



Angle addition postulate

Substitute $x + 28$ for $m\angle SQT$ and $6x - 15$ for $m\angle TQR$.

15. **Example** – In the figure at the right, line MN and line OP intersect at D. Find the value of x and $m\angle ODN$.



Since $\angle MDP$ and $\angle PDN$ form a linear pair, the sum of their measures is 180.

$$m\angle MDP + m\angle PDN = 180$$

$$(9x - 4) + (4x - 11) = 180$$

$$13x - 15 = 180$$

$$13x = 195$$

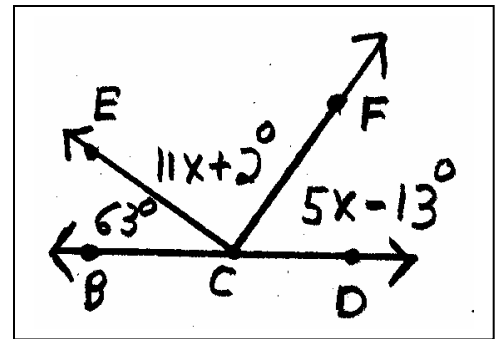
$$x = 15$$

Substitute $9x - 4$ for $\angle MDP$ and $4x - 11$ for $\angle PDN$

Since $\angle MDP$ and $\angle PDN$ are vertical angles, $\angle MDP \cong \angle PDN$

$$\angle PDN = (9(15) - 4)^\circ \rightarrow m\angle PDN = 131^\circ$$

16. **Example** – In the figure at the right, $\angle BCE$ and $\angle ECD$ are supplementary, find $m\angle ECF$ and $m\angle FCD$.



Since $\angle BCE$ and $\angle ECD$ are supplementary, the sum of their measures is 180.

$$m\angle BCE + m\angle ECD = 180$$

$$m\angle ECD = (11x + 2) + (5x - 13) = 16x - 11$$

$$63 + (16x - 11) = 180$$

$$16x = 128$$

$$x = 8$$

Definition supplementary angles

Substitution

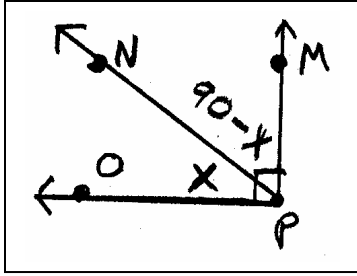
Angle addition postulate

$$\angle ECF = 11(8) + 2 \rightarrow 90^\circ$$

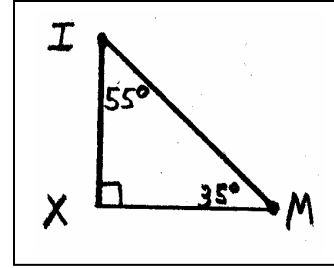
$$\angle FCD = 5(8) - 13 \rightarrow 27^\circ$$

17. **Definition of Complementary Angles** – In the above example, $\angle SQT$ and $\angle TQR$ is 90. If the sum of the measures of two angles is 90, the angles are called complementary angles. Whenever two angles are complementary, each angle is said to be the complement of the other angle.

18. Examples of complementary angles:



$\angle NPO$ is a complement of $\angle NPM$.
 $\angle NPM$ is a complement of $\angle NPO$.



$\angle I$ and $\angle M$ are complementary

19. **Example** – The measure of a supplement of an angle is 2.5 times as large as the measure of a complement of the angle. Find the measure of the angle. Use the 4-step approach to problem solving.

Explore → Let $x =$ the measure of the angle.
 Then $180 - x =$ the measure of its supplement, and
 $90 - x =$ the measure of its complement.

Plan → Write an equation that represents the relationship in this problem.

Supplement	is	2.5 times	the complement
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$$180 - x = 2.5 X (90 - x)$$

Solve → $180 - x = 2.5(90 - x)$
 $180 - x = 225 - 2.5x$
 $1.5x = 45$
 $x = 30$

The measure of the angle is 30.

Examine → Is the measure of the supplement of a 30° angle 2.5 times as large as the measurement of its complement?

Measure of supplement: $180 - 30 = 150$
 Measure of complement: $90 - 30 = 60$
 Since $2.5(60) = 150$, the solution is correct.

Name: _____

Date: _____

Class: _____

Pairs of Angles Worksheet

Answer the following questions.

1. Give an explanation for why you think a linear pair of angles is called “linear.”
2. True or false: All pairs of supplementary angles are also linear pairs. Explain.
3. Complete: Adjacent angles are angles in the same plane that have a common _____ and _____ but no common _____.
4. Describe in your own words the difference between complementary and supplementary angles.

Find the measure of the complement and the supplement of an angle having the indicated measures.

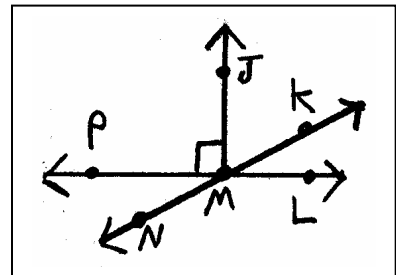
5. 38

7. 110

6. 63

8. x

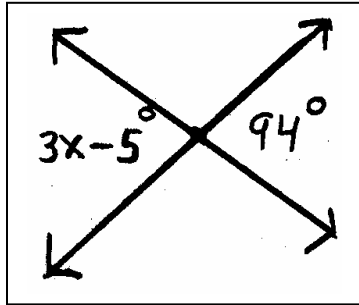
Refer to the figure at the right to name the following pairs of angles (questions 9 thru 14).



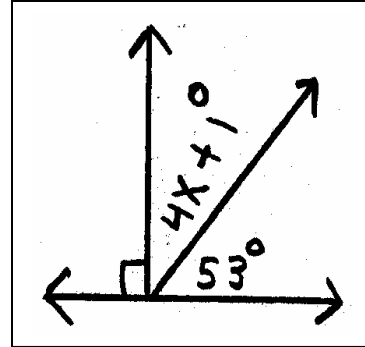
9. Obtuse vertical angles
10. Adjacent complementary angles
11. Congruent supplementary angles
12. Noncongruent supplementary angles
13. Adjacent angles that do not form a linear pair
14. Nonadjacent complementary angles
15. Suppose $\angle A$ is a complement of $\angle B$. Find the value of x , $m\angle A$, and $m\angle B$ if $m\angle A = 7x + 4$ and $m\angle B = 4x + 9$.
16. Suppose $\angle P$ is a supplement of $\angle Q$. Find the value of x , $m\angle P$, and $m\angle Q$ if $m\angle P = 6x + 4$ and $m\angle Q = 10x$.

Find the value of x (questions 17 thru 19).

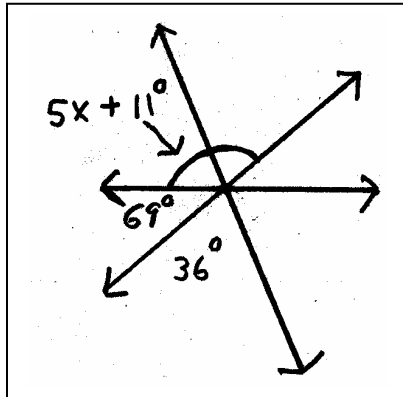
17.



18.

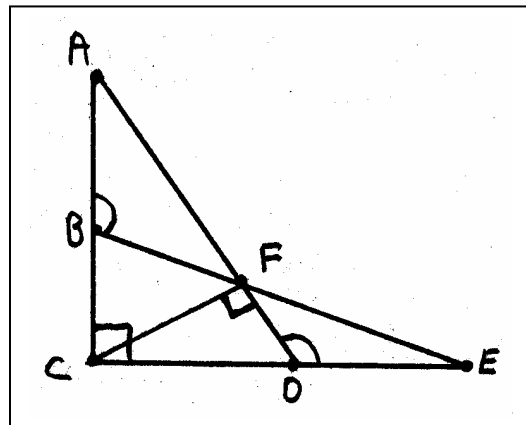


19.



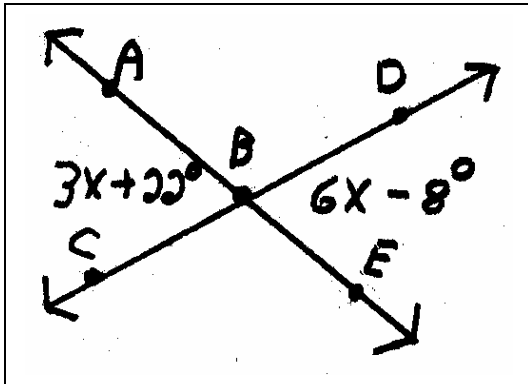
Identify each pair of angles as adjacent, vertical, complementary, supplementary, and/or as a linear pair (questions 20 thru 25).

- 20. $\angle CFE$ and $\angle AFC$
- 21. $\angle BCF$ and $\angle FCD$
- 22. $\angle AFE$ and $\angle DFB$
- 23. $\angle CBF$ and $\angle ABF$
- 24. $\angle AFB$ and $\angle CFB$
- 25. $\angle CBF$ and $\angle FDE$

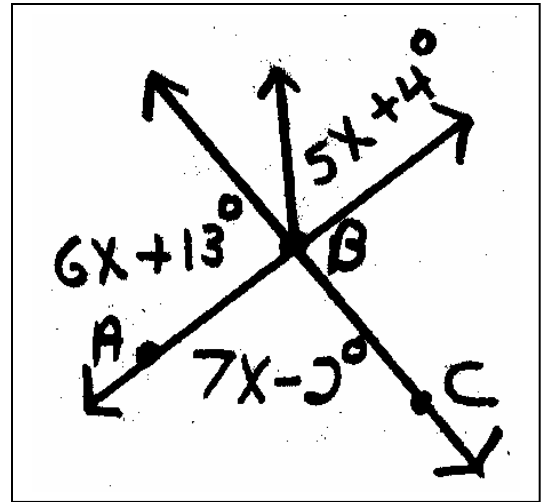


Find the value of x and $m\angle ABC$ (questions 26 thru 28).

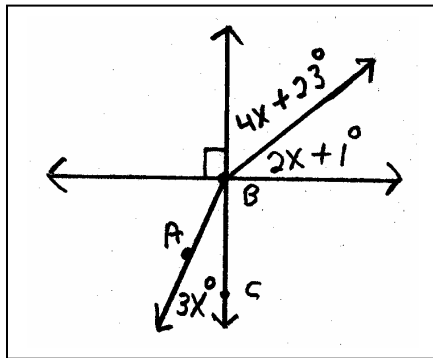
26.



27.



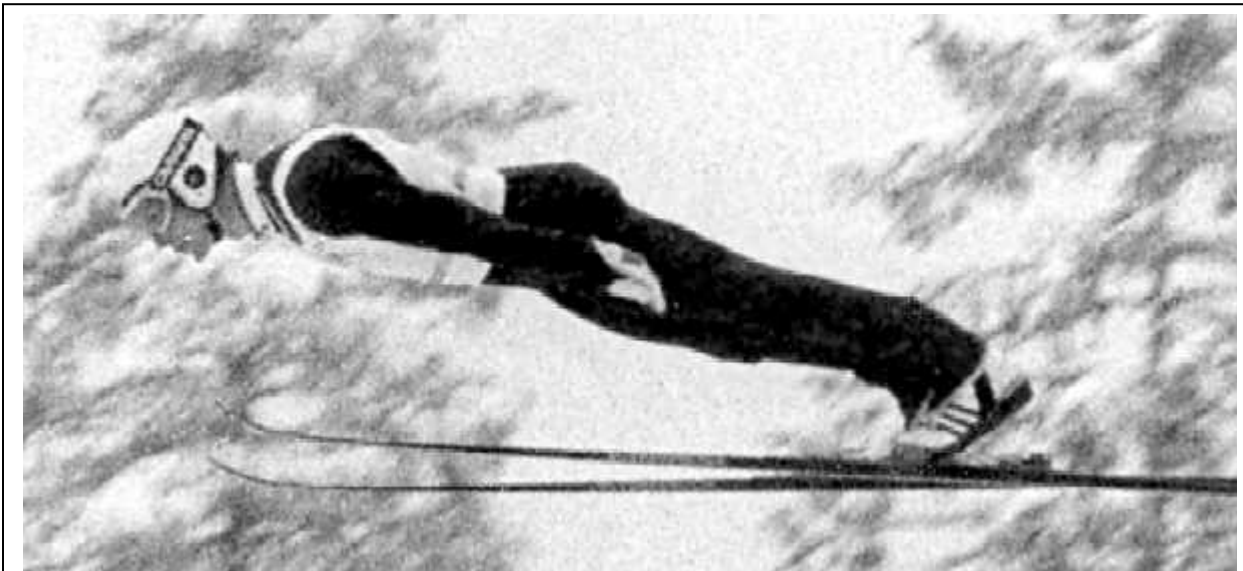
28.



Write and equation and then solve

29. Find the measures of two supplementary angles if the measure of the larger angle is 44 more than the measure of the smaller.
30. Find the measure of two complementary angles if the difference in the measures of the two angles is 12.
31. The measure of an angle is one-third the measure of its supplement. Find the measure of the angle.
32. The measure of an angle is one-fourth the measure of its complement. Find the measure of the angle.
33. The measure of an angle is 6 more than twice the measure of its complement. Find the measures of both angles.

34. The measure of an angle is 5 less than four times the measure of its supplement. Find the measure of both angles.
35. The measure of the supplement of an angle is 6 times the measure of the complement. Find the measures of the angle, its supplement, and its complement.
36. Suppose $\angle X$ and $\angle Y$ are supplementary angles. If $m\angle X = x^2 - 9x$ and $m\angle Y = 11x + 12$, find the value of x , $m\angle X$, and $m\angle Y$.
37. In order for ski jumpers to achieve the maximum distance on a jump, they need to make the angle between their body and the front of their skis as small as possible. This allows them to get the proper extension over the tips of their skis. If a ski jumper's body is aligned so that the angle between the body and the front of the skis is 10° , what will be the angle that the tail of the skis forms with the body?



Name: _____

Date: _____

Class: _____

Pairs of Angles Worksheet Key

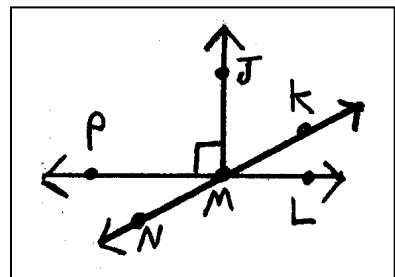
Answer the following questions.

1. Give an explanation for why you think a linear pair of angles is called “linear.” → **Two sides of the pair of angles form a line.**
2. True or false: All pairs of supplementary angles are also linear pairs. Explain. → **False; any two angles whose measures have a sum of 180 are supplementary.**
3. Complete: Adjacent angles are angles in the same plane that have a common _____ and _____ but no common _____. → **vertex; side; interior points**
4. Describe in your own words the difference between complementary and supplementary angles. → **The sum of the measures of complementary angles is 90, while the sum of the measures of supplementary angles is 180.**

Find the measure of the complement and the supplement of an angle having the indicated measures.

5. 38 → $(90 - 38) = 52^\circ$; $(180 - 38) = 142^\circ$
6. 63 → $(90 - 63) = 27^\circ$; $(180 - 63) = 117^\circ$
7. 110 → **no complement**; $(180 - 110) = 70^\circ$
8. x → $(90 - x)^\circ$; $(180 - x)^\circ$

Refer to the figure at the right to name the following pairs of angles (questions 9 thru 14).



9. Obtuse vertical angles → $\angle NML$, $\angle PMK$
10. Adjacent complementary angles → $\angle JMK$, $\angle KML$
11. Congruent supplementary angles → $\angle JMP$, $\angle JML$
12. Noncongruent supplementary angles → **Sample answers** $\angle NMP$ and $\angle PMK$; $\angle NMJ$ and $\angle JMK$; $\angle NML$ and $\angle KML$
13. Adjacent angles that do not form a linear pair → **Sample answers** $\angle NMP$ and $\angle PMJ$; $\angle PMJ$ and $\angle JMK$; $\angle JMK$ and $\angle KML$
14. Nonadjacent complementary angles → $\angle JMK$ and $\angle NMP$

15. Suppose $\angle A$ is a complement of $\angle B$. Find the value of x , $m\angle A$, and $m\angle B$ if $m\angle A = 7x + 4$ and $m\angle B = 4x + 9$.

$$\angle A + \angle B = 90 \rightarrow (7x + 4) + (4x + 9) = 90$$

$$11x + 13 = 90$$

$$11x = 77$$

$$x = 7$$

$$\angle A = 7(7) + 4 = 53^\circ$$

$$\angle B = 4(7) + 9 = 37^\circ$$

16. Suppose $\angle P$ is a supplement of $\angle Q$. Find the value of x , $m\angle P$, and $m\angle Q$ if $m\angle P = 6x + 4$ and $m\angle Q = 10x$.

$$\angle P + \angle Q = 180 \rightarrow (6x + 4) + (10x) = 180$$

$$16x + 4 = 180$$

$$16x = 176$$

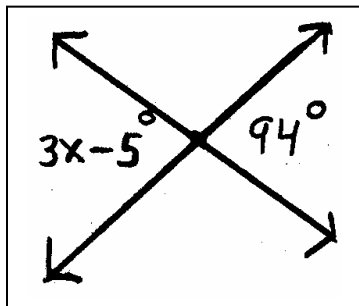
$$x = 11$$

$$\angle P = 6(11) + 4 = 70^\circ$$

$$\angle Q = 10(11) = 110^\circ$$

Find the value of x (questions 17 thru 19).

17.

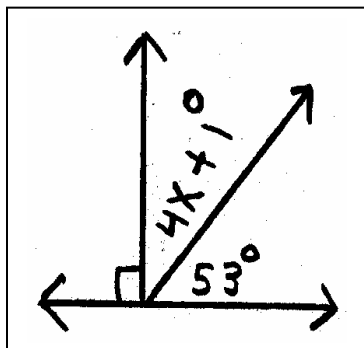


$$3x - 5 = 94$$

$$3x = 99$$

$$x = 33$$

18.

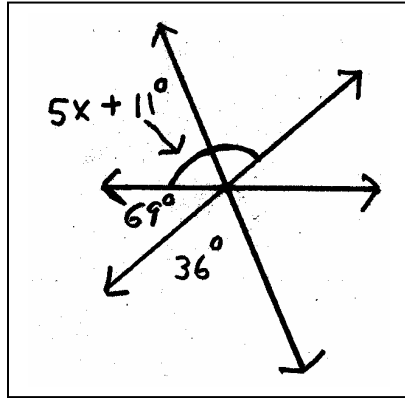


$$4x + 1 + 53 = 90$$

$$4x = 36$$

$$x = 9$$

19.

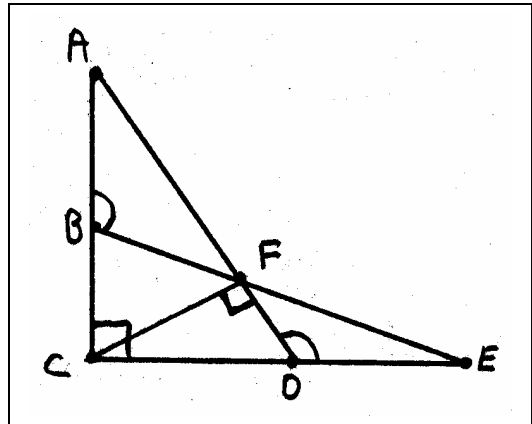


$$(5x + 11) + 69 = 180$$

$$5x = 100$$

$$x = 20$$

Identify each pair of angles as adjacent, vertical, complementary, supplementary, and/or as a linear pair (questions 20 thru 25).



20. $\angle CFE$ and $\angle AFC \rightarrow$ adjacent

21. $\angle BCF$ and $\angle FCD \rightarrow$ adjacent, complementary

22. $\angle AFE$ and $\angle DFB \rightarrow$ vertical

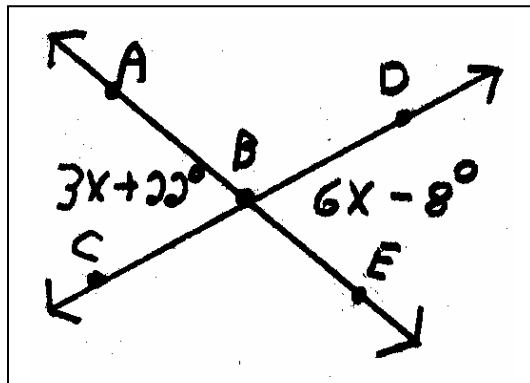
23. $\angle CBF$ and $\angle ABF \rightarrow$ adjacent, supplementary, linear pair

24. $\angle AFB$ and $\angle CFB \rightarrow$ adjacent, complementary

25. $\angle CBF$ and $\angle FDE \rightarrow$ supplementary

Find the value of x and $m\angle ABC$ (questions 26 thru 28).

26.



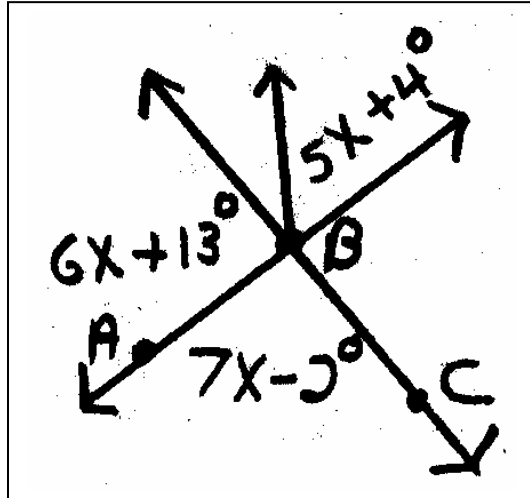
$$3x + 22 = 6x - 8$$

$$30 = 3x$$

$$x = 10$$

$$\angle ABC = 3(10) + 22 \rightarrow 52^\circ$$

27.



$$(6x + 13) + (5x + 4) = 180$$

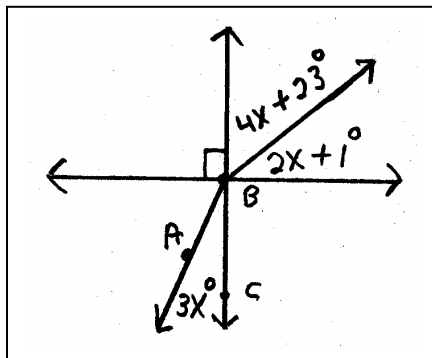
$$11x + 17 = 180$$

$$11x = 163$$

$$x = 14.8$$

$$\angle C = 5(14.8) + 4 \rightarrow 79.4^\circ$$

28.



$$(4x + 23) + (2x + 1) = 180$$

$$6x + 24 = 180$$

$$6x = 156$$

$$\angle C = 2(26) + 1 = 53^\circ$$

Write an equation and then solve

29. Find the measures of two supplementary angles if the measure of the larger angle is 44 more than the measure of the smaller.

$$(x) + (x + 44) = 180$$

$$2x = 136$$

$$x = 68^\circ$$

$$x + 44$$

$$68 + 44$$

$$112^\circ$$

30. Find the measure of two complementary angles if the difference in the measures of the two angles is 12.

$$(x) + (x - 12) = 90$$

$$2x = 102$$

$$x = 51^\circ$$

$$x - 12$$

$$51 - 12$$

$$39^\circ$$

31. The measure of an angle is one-third the measure of its supplement. Find the measure of the angle.

$$x = \text{angle}$$

$$(180 - x) = \text{supplement}$$

$$x = \frac{1}{3}(180 - x)$$

$$3x = 180 - x$$

$$4x = 180$$

$$x = 45^\circ$$

32. The measure of an angle is one-fourth the measure of its complement. Find the measure of the angle.

$$x = \text{angle}$$

$$(90 - x) = \text{complement}$$

$$x = \frac{1}{4}(90 - x)$$

$$4x = 90 - x$$

$$5x = 90$$

$$x = 18^\circ$$

33. The measure of an angle is 6 more than twice the measure of its complement.
Find the measures of both angles.

$$\begin{aligned} x &= \text{measure of angle} \\ 90 - x &= \text{measure of complement} \\ x &= 2(90 - x) + 6 \\ x &= 186 - 2x \\ 3x &= 186 \\ x &= 62^\circ \end{aligned}$$

$$\begin{aligned} 90 - x \\ 90 - 62 \\ 28^\circ \end{aligned}$$

34. The measure of an angle is 5 less than four times the measure of its supplement.
Find the measure of both angles.

$$\begin{aligned} x &= \text{measure of angle} \\ 180 - x &= \text{measure of supplement} \\ x &= 4(180 - x) - 5 \\ x &= 715 - 4x \\ 5x &= 715 \\ x &= 143^\circ \end{aligned}$$

$$\begin{aligned} 180 - x \\ 180 - 143 \\ 37^\circ \end{aligned}$$

35. The measure of the supplement of an angle is 6 times the measure of the complement. Find the measures of the angle, its supplement, and its complement.

$$\begin{aligned} x &= \text{measure of angle} \\ 90 - x &= \text{measure of complement} \\ 180 - x &= \text{measure of supplement} \\ (180 - x) &= 6(90 - x) \\ 180 - x &= 540 - 6x \\ 5x &= 360 \rightarrow x = 72^\circ \end{aligned}$$

$$\begin{aligned} \text{Complement} &= 90 - x = 90 - 72 = 18^\circ \\ \text{Supplement} &= 180 - x = 180 - 72 = 108^\circ \end{aligned}$$

36. Suppose $\angle X$ and $\angle Y$ are supplementary angles. If $m\angle X = x^2 - 9x$ and $m\angle Y = 11x + 12$, find the value of x , $m\angle X$, and $m\angle Y$.

$$\angle X + \angle Y = 180$$

$$(x^2 - 9x) + (11x + 12) = 180$$

$$x^2 + 2x - 168 = 0$$

$$(x - 12)(x + 14) = 0$$

$$x = 12 \text{ or } x = -14$$

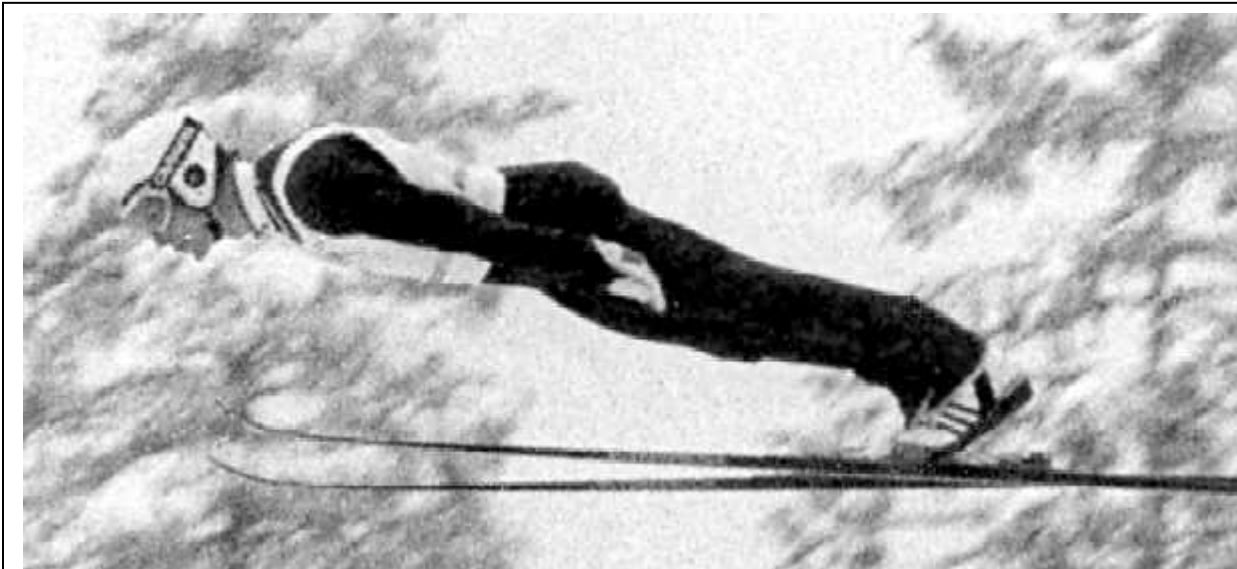
-14 cannot be a solution because
 $\angle Y = 11(-14) + 12 = -142$

$$m\angle X = x^2 - 9x \rightarrow (12)^2 - 9(12) = 36^\circ$$

$$m\angle Y = 11x + 12 \rightarrow 11(12) + 12 = 144^\circ$$

37. In order for ski jumpers to achieve the maximum distance on a jump, they need to make the angle between their body and the front of their skis as small as possible. This allows them to get the proper extension over the tips of their skis. If a ski jumper's body is aligned so that the angle between the body and the front of the skis is 10° , what will be the angle that the tail of the skis forms with the body?

Hint: Think about the relationship between the ski jumper's body and the skis.
 170°



Student Name: _____

Date: _____

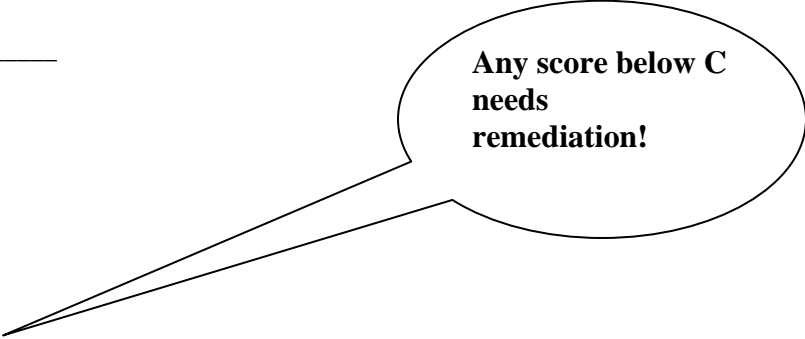
Pairs of Angles Checklist

1. On questions 1 thru 4 , did the student answer questions correctly?
 - a. Yes (20 points)
 - b. 3 out of 4 (15 points)
 - c. 2 out of 4 (10 points)
 - d. 1 out of 4 (5 points)
2. On questions 5 thru 8, did the student find the measure of the complement correctly?
 - a. Yes (20 points)
 - b. 3 out of 4 (15 points)
 - c. 2 out of 4 (10 points)
 - d. 1 out of 4 (5 points)
3. On questions 5 thru 8, did the student find the measure of the supplement correctly?
 - a. Yes (20 points)
 - b. 3 out of 4 (15 points)
 - c. 2 out of 4 (10 points)
 - d. 1 out of 4 (5 points)
4. On questions 9 thru 14, did the student answer questions correctly?
 - a. All (30 points)
 - b. 5 out of 6 (25 points)
 - c. 4 out of 6 (20 points)
 - d. 3 out of 6 (15 points)
 - e. 2 out of 6 (10 points)
 - f. 1 out of 6 (5 points)
5. On question 15, did the student find the missing measures?
 - a. All (15 points)
 - b. 2 out of 3 (10 points)
 - c. 1 out of 3 (5 points)
6. On question 16, did the student find the missing measures?
 - a. All (15 points)
 - b. 2 out of 3 (10 points)
 - c. 1 out of 3 (5 points)
7. On questions 17 thru 19, did the student find the value of x?
 - a. All (15 points)
 - b. 2 out of 3 (10 points)
 - c. 1 out of 3 (5 points)
8. On questions 20 thru 25, did the student identify the angle pairs correctly?
 - a. Yes (30 points)
 - b. 5 out of 6 (25 points)
 - c. 4 out of 6 (20 points)
 - d. 3 out of 6 (15 points)
 - e. 2 out of 6 (10 points)
 - f. 1 out of 6 (5 points)
9. On questions 26 thru 28, did the student find the value of x correctly?
 - a. All (15 points)
 - b. 2 out of 3 (10 points)
 - c. 1 out of 3 (5 points)
10. On questions 26 thru 28, did the student find the measure of $\angle ABC$ correctly?
 - a. All (15 points)
 - b. 2 out of 3 (10 points)
 - c. 1 out of 3 (5 points)

11. On question 29, did the student find the measure of both angles correctly?
- a. Yes (10 points)
 - b. Found the value of one angle (5 points)
12. On question 30, did the student find the measure of both angles correctly?
- a. Yes (10 points)
 - b. Found the value of one angle (5 points)
13. On questions 31 and 32, did the student find the measure of the angle correctly?
- a. Yes (10 points)
 - b. Found the value of one angle (5 points)
14. On question 33, did the student find the measure of both angles correctly?
- a. Yes (10 points)
 - b. Found the value of one angle (5 points)
15. On question 34, did the student find the measure of both angles correctly?
- a. Yes (10 points)
 - b. Found the value of one angle (5 points)
16. On question 35, did the student find all missing measures correctly?
- a. Yes (15 points)
 - b. 2 out of 3 (10 points)
 - c. 1 out of 3 (5 points)
17. On question 36, did the student find all missing measures correctly?
- a. Yes (15 points)
 - b. 2 out of 3 (10 points)
 - c. 1 out of 3 (5 points)
18. On question 37, did the student answer the question correctly?
- a. Yes (5 points)

Total Number of Points _____

- A 162 points and above
- B 144 points and above
- C 126 points and above
- D 108 points and above
- F 107 points and below



**Any score below C
needs
remediation!**