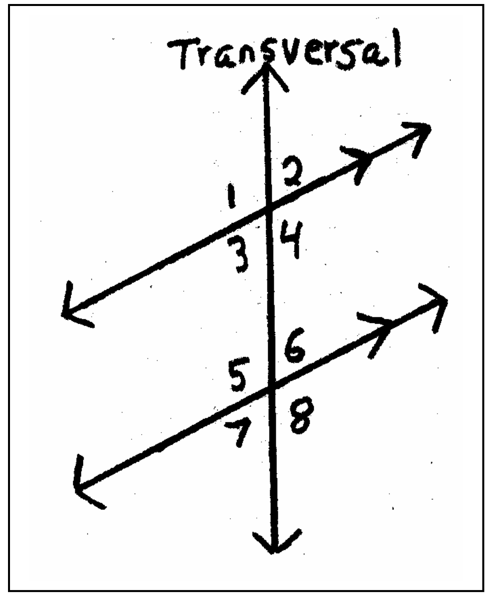
Determining Angle Measure with Parallel Lines Examples

1. Using the figure at the right, review with students the following angles: corresponding, alternate interior, alternate exterior and consecutive interior.



Corresponding Angles

$\angle 1$ and $\angle 5$, $\angle 3$ and $\angle 7$, $\angle 2$ and $\angle 6$, $\angle 4$ and $\angle 8$

Alternate Interior Angles

$\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$

Alternate Exterior Angles

$\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 7$

Consecutive Interior Angles

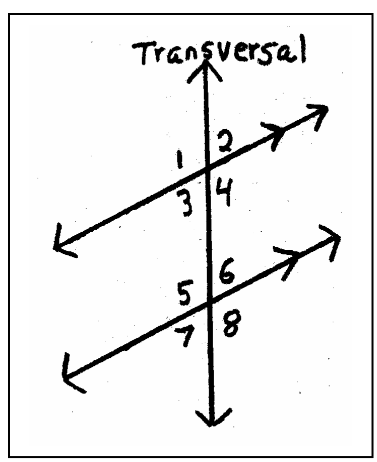
$\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$

2. There are several postulates and theorems that help you gain insight into properties of parallel lines. If you investigate the relationships between the corresponding angles formed by two parallel lines cut by a transversal, you will observe that the angles are congruent. This property is accepted as a postulate.

3. **Corresponding Angles Postulate –** If two parallel lines are cut by a transversal, then each pair of corresponding angles is congruent.

This postulate, combined with linear pair and vertical angle properties, helps to establish several angle relationships.

4. **Example –** A road crosses a set of railroad tracks. If the measure of $\angle 6$ is 110^{o}, find m$\angle 3$.



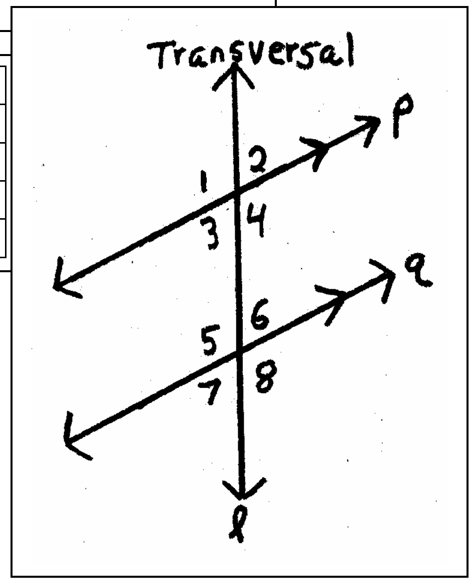
Since $\angle 2$ and $\angle 6$ are corresponding angles, m$\angle 2$ = m$\angle 6$, m$\angle 2$ =m$\angle 3$ because they are vertical angles. Therefore, m$\angle 6$ = m$\angle 3$ by the transitive property of equality. So, m$\angle 3$ = 110^{o}.

5. Notice that in Example 4 above, ∠4 and ∠5 form a pair of alternate interior angles. This is an application of another of the special relationships between the angles formed by two parallel lines and a transversal. These relationships are summarized in the theorems below.

6. **Alternate Interior Angle Theorem –** If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.

The proof of the Alternate Interior Angle Theorem is given below.

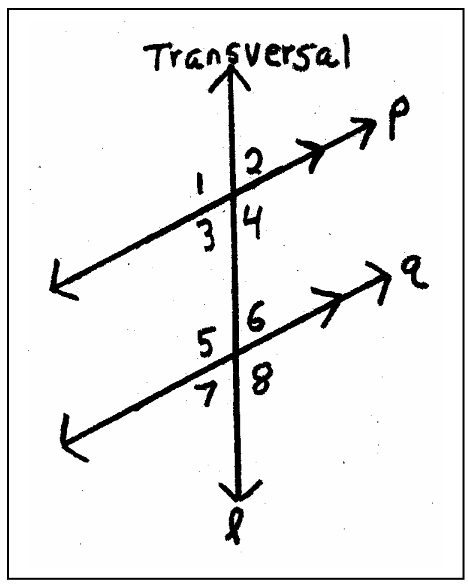
Statement	Reason
P||q and transversal l	Given
∠3 ≅ ∠2	Vertical Angle Theorem
∠2 ≅ ∠6	Corresponding Angle Theorem
∠3 ≅ ∠6	Transitive Property



7. **Consecutive Interior Angle Theorem –** If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.

The proof of the Alternate Exterior Angle Theorem is given below.

Statement	Reason
P||q and transversal l	Given
∠1 ≅ ∠5	Corresponding Angle Theorem
∠5 ≅ ∠8	Vertical Angle Theorem
∠1 ≅ ∠8	Transitive Property

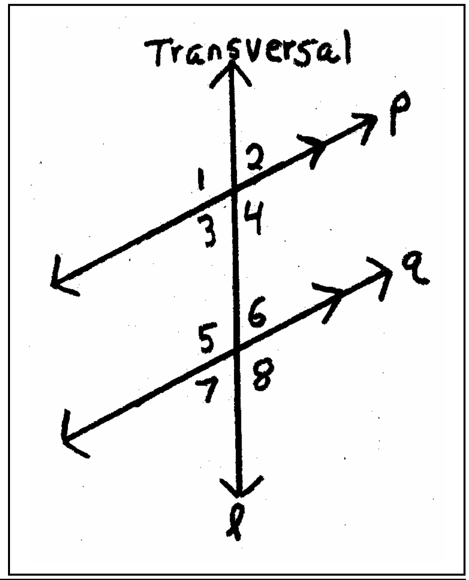


8. **Alternate Exterior Angle Theorem** – If two lines are cut by a transversal, then each pair of alternate exterior angles is congruent.

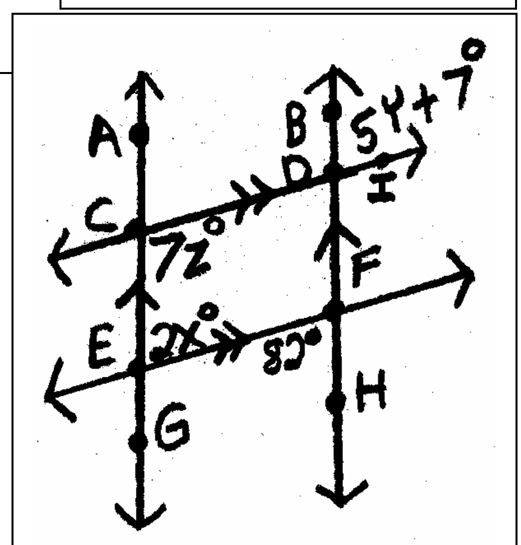
The proof of the Alternate Exterior Angle Theorem is given below in a paragraph proof. The statements and reasons are written informally in a paragraph. But the steps in a paragraph proof are the same as those in a two-column proof.

Given: $\ell \parallel q$ **and** ℓ **is a transversal of p and q.**
Prove: $\angle 1 \cong \angle 8$; $\angle 2 \cong \angle 7$

Paragraph Proof:
We are given that p || q. If two parallel lines are cut by a transversal, corresponding angles are congruent. So, $\angle 1 \cong \angle 5$ and $\angle 2 \cong \angle 6$. $\angle 5 \cong \angle 8$ and $\angle 6 \cong \angle 7$ because vertical angles are congruent. Therefore, $\angle 1 \cong \angle 8$ and $\angle 2 \cong \angle 7$ since congruence of angles is transitive.



9. **Example** – Find the values of x, y, and z.



Since $\overleftrightarrow{AG} \parallel \overleftrightarrow{BH}$, $\angle$CEF $\cong$ $\angle$EFH by the alternate interior angle theorem.

m$\angle$CEF = m$\angle$EFH
2x = 82
x = 41

Since $\overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$, $\angle$DCE and $\angle$CEF are supplementary by the consecutive interior angle theorem.

m$\angle$DCE +m$\angle$CEF = 180
7z + 2x = 180
7z + 2(41) = 180
7z = 98
z = 14

Since $\overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$, $\angle$BDI $\cong$ $\angle$EFH by the alternate exterior angle theorem.

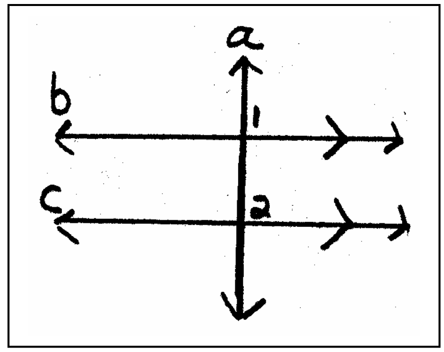
m$\angle$BDI = m$\angle$EFH
5y + 7 = 82
5y = 75
y = 15

Therefore, x = 41, y = 15, and z= 14

10. **Perpendicular Transversal Theorem –** In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other.

The proof of the Perpendicular Transversal Theorem is given below.

$\overleftrightarrow{b} \parallel \overleftrightarrow{c}$ **and** $\overleftrightarrow{a} \perp \overleftrightarrow{b}$	**Given**
$\angle$**1 is a 90°angle**	**Definition of Perpendicular Lines**
$\angle 1 \cong \angle 2$	**Parallel Lines Postulate**
m$\angle$**2 is 90°**	**Transitive Property**
$\overleftrightarrow{a} \perp \overleftrightarrow{c}$	**Definition of Perpendicular Lines**



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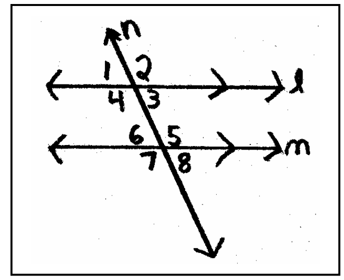
Class:____________________

Determining Angle Measure with Parallel Lines Worksheet

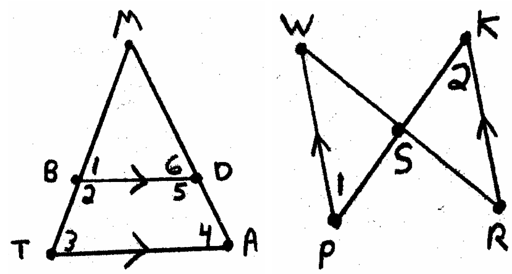
Answer the following questions.



1. Explain why $\angle 4$ and $\angle 6$ must be supplementary.

2. If you know that $m\angle 1 = 70$, explain two different strategies you could use to find $m\angle 5$.

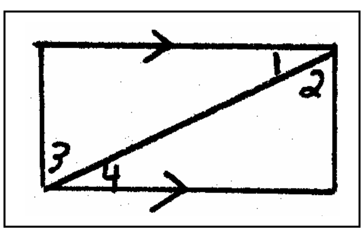
3. Explain what the arrowheads on the lines in both diagrams at the right indicate.

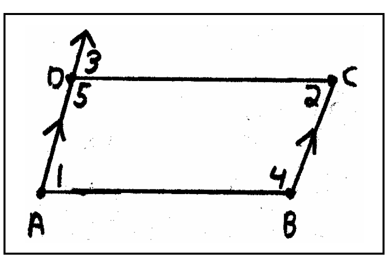


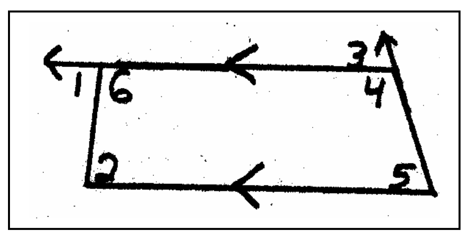
4. If $\overleftrightarrow{BD} \parallel \overleftrightarrow{AT}$, then $\angle 1 \cong \angle 3$ and $\angle 6 \cong \angle 4$. Explain why this is true. (see figure above)

5. If $\overleftrightarrow{WP} \parallel \overleftrightarrow{KR}$, then $\angle 1 \cong \angle 2$. Explain why this is true. (see figure above)

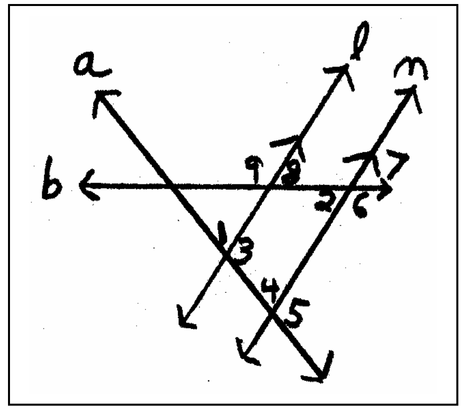
List the conclusions that can be drawn from each figure.

6. 

7. 

8. 

Given $\ell \parallel m$, m∠1 = 98, and m∠2 = 40, find the measure of each angle.



9. m∠3

10. m∠4

11. m∠5

12. m∠6

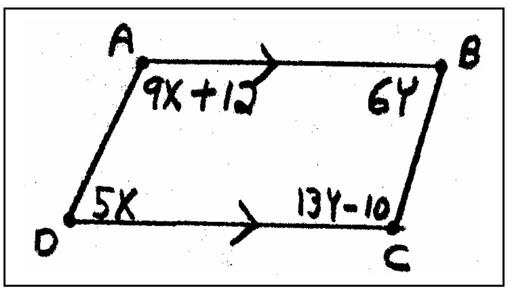
13. m∠7

14. m∠8

15. m∠9

Find the values of x and y

16.



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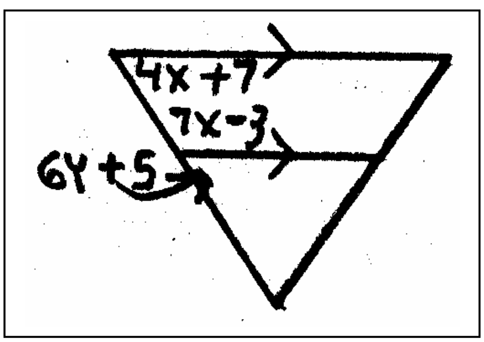

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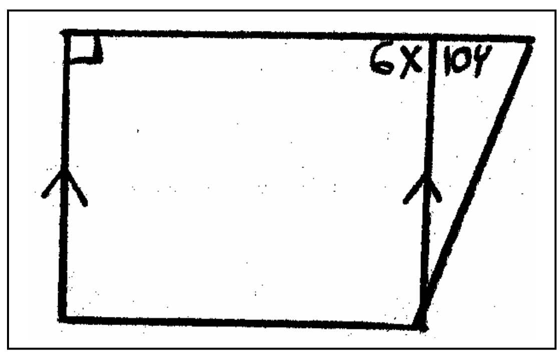
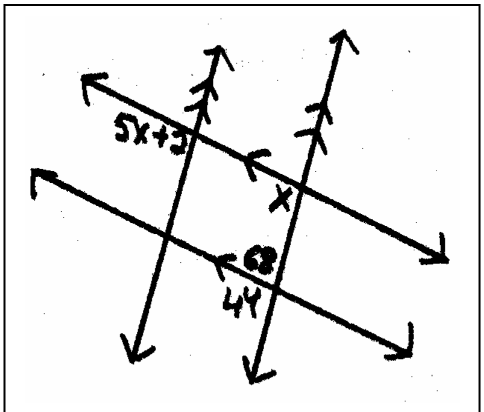


Find the values of x, y and z

19.



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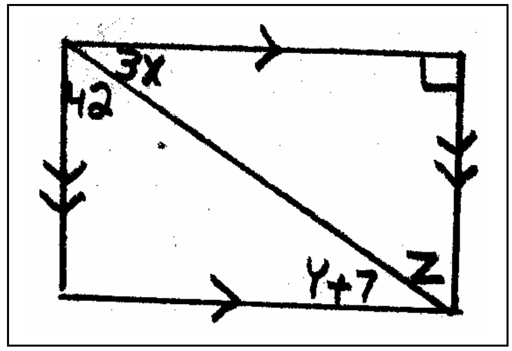

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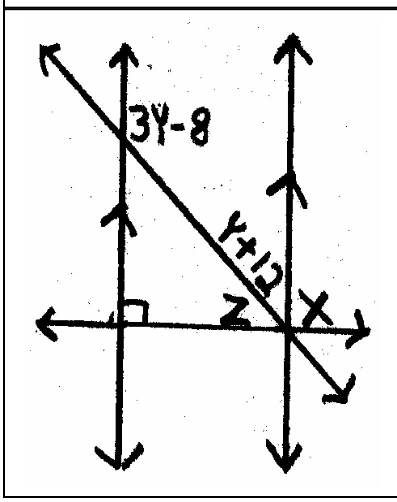
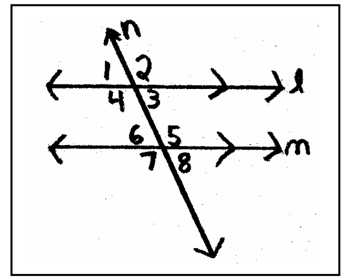


Name:___________________
Date:____________
Class:___________________

Determining Angle Measure with Parallel Lines Worksheet Key

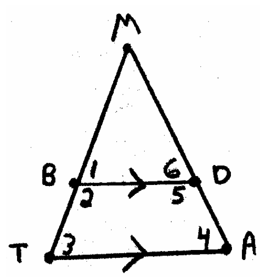
Answer the following questions.

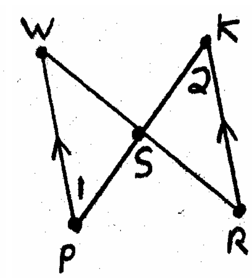


1. Explain why $\angle 4$ and $\angle 6$ must be supplementary. ➔**If 2 parallel lines are cut by a transversal, consecutive interior angles are supplementary.**

2. If you know that m$\angle 1 = 70$, explain two different strategies you could use to find m$\angle 5$. ➔**Sample answers: First m$\angle 1$ = m$\angle 3$ because $\angle 1$ and $\angle 3$ are vertical angles, then m$\angle 3$ + m$\angle 5$ = 180 because, if 2 parallel lines re cut by a transversal, consecutive interior angles are supplementary. Therefore, 70 + m$\angle 5$ = 180 and m$\angle 5$ = 110. m$\angle 1$ = m$\angle 8$ because if 2 parallel lines are cut by a transversal, alternate exterior angles are congruent. $\angle 5$ and $\angle 8$ are supplementary because $\angle 5$ and $\angle 8$ are a linear pair and if two angles form a linear pair, they are supplementary. So m$\angle 5$ and m$\angle 8$ = 180 by definition of supplementary angles. Therefore, m$\angle 5$ + 70 = 180 and m$\angle 5$ = 110.**

3. Explain what the arrowheads on the lines in both diagrams at the right indicate. ➔**Lines are parallel**



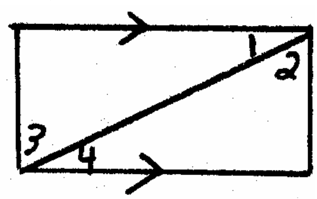


4. If $\overleftrightarrow{BD} \parallel \overleftrightarrow{AT}$, then $\angle 1 \cong \angle 3$ and $\angle 6 \cong \angle 4$. Explain why this is true. (see figure above) ➔ **If 2 lines are cut by a transversal, corresponding angles are congruent.**

5. If $\overleftrightarrow{WP} \parallel \overleftrightarrow{KR}$, then $\angle 1 \cong \angle 2$. Explain why this is true. (see figure above) ➔ **If 2 lines are cut by a transversal, alternate interior angles are congruent.**

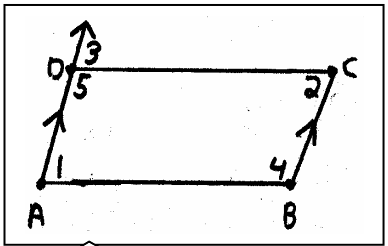
List the conclusions that can be drawn from each figure.

6.



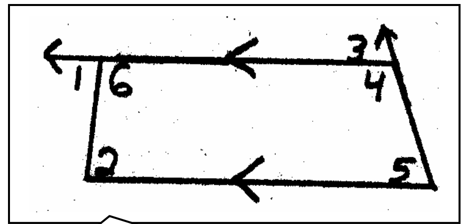
➔ **$\angle 1 \cong \angle 4$**

7.



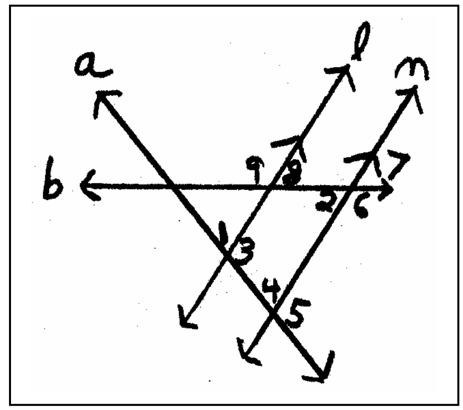
∠5 and ∠2 are supplementary; ∠1 and ∠4 are supplementary; ∠3 ≅ ∠2; ∠3 and ∠5 are supplementary.

8.



∠2 and ∠6 are supplementary; ∠4 and ∠5 are supplementary; ∠1 ≅ ∠2; ∠3 ≅ ∠5; ∠3 and ∠4 are supplementary.

Given $\ell \parallel m$, m∠1 = 98, and m∠2 = 40, find the measure of each angle.



9. m∠3 ➔ **(180 – 98) = 82**

10. m∠4 ➔ **98 (corresponding angles)**

11. m∠5 ➔ **(180 – 98) = 82**

12. m∠6 ➔ **(180 – 40) = 140**

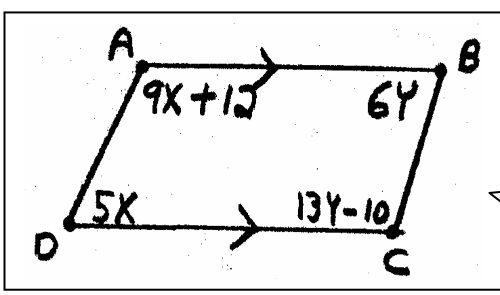
13. m∠7 ➔ **40 (Linear Pair)**

14. m∠8 ➔ **40 (alternate interior angles)**

15. m∠9 ➔ **140 (linear pair)**

Find the values of x and y

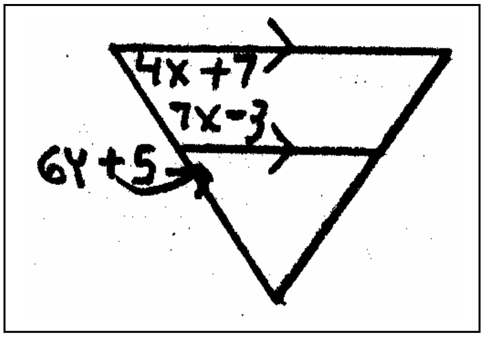
16.



(9x + 12) + (5x) = 180
14x = 168
x = 12

(6y) + (13y – 10) = 180
19y = 190
y = 10

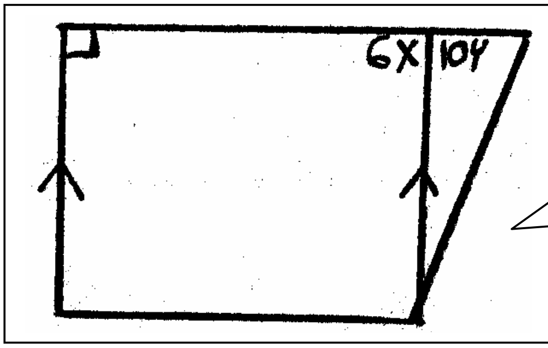
17.



(4x + 7) + (7x – 3) = 180
11x = 176
x = 16

(7(16) – 3) + (6y + 5) = 180
114 + 6y = 180
6y = 66
y = 11

18.

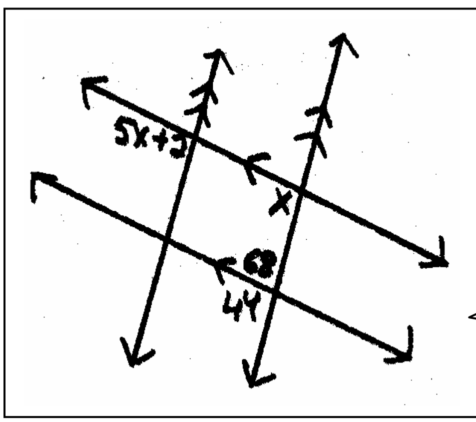


6x = 90
x = 15

10y = 90
y = 9

Find the values of x, y and z

19.



x + 68 = 180
x = 112

4y + 68 = 180
4y = 112
y = 28

5z + 2 = 112
5z = 110
z = 22

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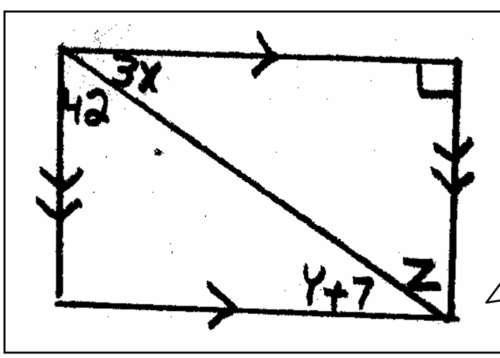
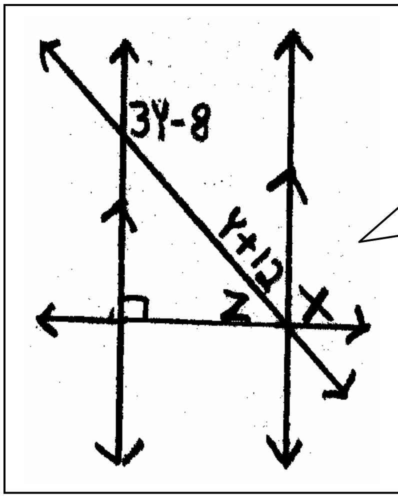


3x + 42 = 90
3x = 38
x = 16

z = 42 (alternate interior angles)

y + 7 = 3(16)   (alternate interior angles)
y = 41

21. 

(3y – 8) + (y + 12) = 180
4y = 176
y = 44

z + (44 + 12) = 90
z = 34

x = 90

Student Name: ___________________

Date: _______________

Determining Angle Measure with Parallel Lines Checklist

1. On question 1, did the student explain why angles are supplementary?

   a. Yes (5 points)

2. On question 2, did the student use two different strategies for finding angle 5?

   a. Yes (10 points)
   b. Used only 1 method (5 points)

3. On questions 3, did the student explain what the arrowheads represented?

   a. Yes (5 points)

4. On questions 4 and 5, did the student explain why the statement was true?

   a. Yes (10 points)
   b. 1 out of 2 (5 points)

5. On questions 6 thru 8, did the student list conclusions from drawings?

   a. All (15 points)
   b. 2 out of 3 (10 points)
   c. 1 out of 3 (5 points)

6. On questions 9 thru 15, did the student find the missing measures?

   a. All (35 points)
   b. 6 out of 7 (30 points)
   c. 5 out of 7 (25 points)
   d. 4 and of 7 (20 points)
   e. 3 out of 7 (15 points)
   f. 2 out of 7 (10 points)
   g. 1 out of 7 (5 points)

7. On questions 16 thru 21, did the student find the value of x?

   a. All (30 points)
   b. 5 out of 6 (25 points)
   c. 4 out of 6 (20 points)
   d. 3 out of 6 (15 points)
   e. 2 out of 6 (10 points)
   f. 1 out of 6 (5 points)

8. On questions 16 thru 21, did the student find the value of y?

   a. All (30 points)
   b. 5 out of 6 (25 points)
   c. 4 out of 6 (20 points)
   d. 3 out of 6 (15 points)
   e. 2 out of 6 (10 points)
   f. 1 out of 6 (5 points)

9. On questions 22 and 21, did the student find the value of z correctly?

   a. All (10 points)
   b. 1 out of 2 (5 points)

Total Number of Points __________

Any score below C needs remediation!

A 141 points and above

B 128 points and above

C 113 points and above

D 90 points and above

F 107 points and below

NOTE: The sole purpose of this checklist is to aide the teacher in identifying students that need remediation. Students who meet the "C" criteria are ready for the next level of learning.