How Do You Measure a Triangle? Examples

1. A **triangle** is a three-sided **polygon**. A **polygon** is a closed figure in a plane that is made up of segments called **sides** that intersect only at their endpoints, called **vertices**.

2. Triangle ABC, written ΔABC, has the following parts.

<table>
<thead>
<tr>
<th>Sides:</th>
<th>Vertices: A, B, C</th>
</tr>
</thead>
<tbody>
<tr>
<td>∆ABC</td>
<td>Angles: ∠BAC, ∠A, ∠ABC, ∠B, ∠BCA</td>
</tr>
</tbody>
</table>

3. The side opposite ∠A is BC. The angle opposite AB is ∠C, What side is opposite ∠B? \( AC \).

4. One way of classifying triangles is by their angles. All triangles have at least two acute angles, but the third angle may be acute, right, or obtuse. A triangle can be classified using the third angle.

   - In an acute triangle, all the angles are acute.
   - In an obtuse triangle, one angle is obtuse.
   - In a right triangle, one angle is a right angle.

5. When all of the angles of a triangle are congruent, the triangle is **equiangular**.
6. Provide students with strips of heavy paper and paper fasteners. Ask them to use these to investigate the following:
   a. Can a triangle include a right angle and an obtuse angle? \( \rightarrow \) No
   b. Can a triangle include two obtuse angles? \( \rightarrow \) No
   c. If a triangle includes a right angle, what must be true of the other angles? \( \rightarrow \) Both acute
7. Some parts of a right triangle have special names. In right triangle RST, \( \overline{RT} \), the side opposite the right angle, is called the hypotenuse. The other two sides, \( \overline{RS} \) and \( \overline{ST} \), are called the legs.
8. **Example** – The incline shown at the right is a right triangle. If the vertices B, L, and T are labeled as shown, name the angles, the right angle, the hypotenuse, the legs, the side opposite \( \angle B \), and the angle opposite \( \overline{BL} \).

The angles are \( \angle B \), \( \angle T \), and \( \angle L \). The right angle is \( \angle L \). The hypotenuse is the side opposite the right angle, \( \overline{BT} \). The legs are \( \overline{LB} \) and \( \overline{LT} \). The side opposite \( \angle B \) is \( \overline{LT} \). The angle opposite \( \overline{BL} \) is \( \angle T \).
9. Triangles can also be classified according to the number of congruent sides. The slashes on the sides of a triangle mean those sides are congruent.

No two sides of a scalene triangle are congruent.

At least two sides of an isosceles triangle are congruent.

All the sides of an equilateral triangle are congruent.
10. Like the right triangle, the parts of an isosceles triangle have special names. The congruent sides are called legs. The angle formed by the legs is the **vertex angle**, and the other two angles are **base angles**. The base is the side opposite the vertex angle.
11. **Example** – Triangle ABC is an isosceles triangle. 
∠A is the vertex angle, \( AB = 4x - 14 \) and \( AC = x + 10 \). Find the length of the legs.

If \( \angle A \) is the vertex angle, then \( BC \) is the base angle and \( AB \) and \( AC \) are the legs. So, \( AB = AC \).

Solve the following equation.
\[
AB = AC \\
4x - 14 = x + 10 \\
3x = 24 \\
x = 8
\]

If \( x = 8 \), then \( AB = 4(8) - 14 \) or 18, and \( AC = (8) + 10 \) or 18. The legs of isosceles \( \triangle ABC \) are 18 units long.

12. **Example** – Triangle PQR is an equilateral triangle. One side measures \( 2x + 5 \) and another side measures \( x + 35 \). Find the length of each side.

If \( \triangle PQR \) is equilateral, then each side is congruent.

Solve the following equation.
\[
2x + 5 = x + 35 \\
x + 5 = 35 \\
x = 30
\]

If \( x = 30 \), then \( 2(30) + 5 \) is 65. The length of each side is 65.

13. **Example** – Given \( \triangle MNP \) with vertices \( M(2, -4) \), \( N (-3, 1) \), and \( P(1, 6) \), use the distance formula to prove \( \triangle MNP \) is scalene.

According to the distance formula, the distance between \( (x_1, y_1) \) and \( (x_2, y_2) \) is \( \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \) units.

\[
\begin{align*}
MN &= \sqrt{(2 - (-3))^2 + (-4 - 1)^2} \\
MN &= \sqrt{25 + 25} \Rightarrow \sqrt{50} \text{ or } 5\sqrt{2} \\
NP &= \sqrt{(-3 - 1)^2 + (1 - 6)^2} \\
NP &= \sqrt{16 + 25} \Rightarrow \sqrt{41} \\
MP &= \sqrt{(2 - 1)^2 + (-4 - 6)^2} \\
MP &= \sqrt{1 + 100} \Rightarrow \sqrt{101}
\end{align*}
\]

Since no two sides have the same length, the triangle is scalene.
14. **Angle Sum Theorem**  
The sum of the measures of the angles of a triangle is 180.

In order to prove the angle sum theorem, you need to draw an auxiliary line. An auxiliary line is a line or line segment added to a diagram to help in a proof. These are shown as dashed lines in the diagram. Be sure that it is possible to draw any auxiliary lines that you use.

<table>
<thead>
<tr>
<th>Given: $\triangle PQR$</th>
<th>Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180$</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\triangle PQR$</td>
<td>Given</td>
</tr>
<tr>
<td>Draw $AB$ through $R$ parallel to $PQ$</td>
<td>Parallel Postulate</td>
</tr>
<tr>
<td>$\angle 4$ and $\angle PRB$ form a linear pair</td>
<td>Definition of linear pair</td>
</tr>
<tr>
<td>$\angle 4$ and $\angle PRB$ are supplementary</td>
<td>If 2 $\angle$’s form a linear pair, they are supplementary</td>
</tr>
<tr>
<td>$\angle 4$ and $\angle PRB = 180$</td>
<td>Definition of supplementary angles</td>
</tr>
<tr>
<td>$\angle 5$ and $\angle PRB = 180$</td>
<td>Angle addition postulate</td>
</tr>
<tr>
<td>$m\angle 4 + m\angle 5 + m\angle 3 = 180$</td>
<td>Substitution property of equality</td>
</tr>
<tr>
<td>$m\angle 1 = m\angle 4$</td>
<td>If 2</td>
</tr>
<tr>
<td>$m\angle 2 = m\angle 5$</td>
<td></td>
</tr>
<tr>
<td>$m\angle 1 + m\angle 2 + m\angle 3 = 180$</td>
<td>Substitution property of equality</td>
</tr>
</tbody>
</table>

15. **Example** – The roof support of a building is shaped like a triangle. Two angles each have a measure of 25. Find the measure of the third angle.

Label the vertices of the triangle $P$, $Q$, and $R$. Then, $m\angle P = 25$ and $m\angle Q = 25$. Since the sum of the angles measure is 180, we can write the equation below.

\[
m\angle p + m\angle Q + m\angle R = 180
\]
\[
25 + 25 + m\angle R = 180
\]
\[
m\angle R = 130
\]

The measure of the third angle is 130.

16. **Example** – A surveyor has drawn a triangle on a map. One angle measures 42 and the other measures 53. Find the measure of the third angle.

\[
42 + 53 + x = 180
\]
\[
75 + x = 180
\]
\[
x = 85
\]
17. \[ \angle A \cong \angle D \text{ and } \angle B \cong \angle E \]
Prove: \[ \angle C \cong \angle F \]

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \angle A \cong \angle D \text{ and } \angle B \cong \angle E )</td>
<td>Given</td>
</tr>
<tr>
<td>( m \angle A = m \angle D \text{ and } m \angle B = m \angle E )</td>
<td>Definition of congruent angles</td>
</tr>
<tr>
<td>( m \angle A + m \angle B + m \angle C = 180 )</td>
<td>The sum of the ( \angle )'s in a ( \triangle ) is 180°</td>
</tr>
<tr>
<td>( m \angle D + m \angle E + m \angle F = 180 )</td>
<td>Substitution property of equality</td>
</tr>
<tr>
<td>( m \angle A + m \angle B + m \angle C = m \angle D + m \angle E + m \angle F )</td>
<td>Subtraction property of equality</td>
</tr>
<tr>
<td>( m \angle C = m \angle F )</td>
<td>Definition of congruent angles</td>
</tr>
</tbody>
</table>

18. Suppose you are picking vegetables from a garden and started walking from a certain point on a north-south path. You walked at an angle of 60° northeast for 800 feet, turned directly south and walked 400 feet, and then turned 90° clockwise and walked back to the same place where you started.

A diagram of your path is shown at the right. The north/south lines are parallel, forming congruent alternate interior angles. By the supplement theorem, the angle formed when you turned measures 120°.

The 120° and 90° angles formed when you turned are called exterior angles of the triangle. An exterior angle is formed by one side of a triangle and another side extended. The interior angles of the triangle not adjacent to a given exterior angle are called remote interior angles of the triangle. In the figure at the right, \( \angle BCD \) is an exterior angle with \( \angle A \) and \( \angle B \) as its remote interior angles.

19. How are the measure of an exterior angle and its remote interior angles related?

\[
\begin{align*}
m \angle 1 + m \angle 2 + m \angle 3 &= 180 \\
m \angle 1 + m \angle 4 &= 180 \\
m \angle 1 + m \angle 2 + m \angle 3 &= m \angle 1 + m \angle 4 \\
m \angle 2 + m \angle 3 &= m \angle 4
\end{align*}
\]

Have students make a conjecture about the relationship.
20. **Exterior Angle Theorem**

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.

21. **Example** – Find the measure of each angle in the figure if $\overline{RK} \parallel \overline{SL}$ and $\overline{RS} \perp \overline{SL}$.

By the exterior angle theorem, $m \angle JRK + m \angle 4 = m \angle MKR$.

$m \angle KRL + m \angle 4 = m \angle MKR$
$40 + m \angle 4 = 115$
$m \angle 4 = 75$

By the Angle sum theorem, $m \angle KRL + m \angle 3 + m \angle 4 = 180$

$m \angle KRL + m \angle 3 + m \angle 4 = 180$
$40 + m \angle 3 + 75 = 180$
$m \angle 3 = 65$

Since $\overline{RK} \parallel \overline{SL}$, and $\angle 5$ and $\angle KRL$ are alternate interior angles, $m \angle 5 = m \angle KRL$

$m \angle 5 = 40$

$\overline{RS} \perp \overline{SL}$ and perpendicular lines form right angles. Therefore, $m \angle 1 = 90$.

$m \angle 1 + m \angle 5 + m \angle 2 = 180$
$90 + 40 + m \angle 2 = 180$
$m \angle 2 = 50$

Therefore, $m \angle 1 = 90$, $m \angle 2 = 50$, $m \angle 3 = 65$, $m \angle 4 = 75$, and $m \angle 5 = 40$.

22. A statement that can easily be proven using a theorem is called a **corollary** of that theorem. A **corollary**, just like a **theorem**, can be used as a reason of that proof.
23. **Corollary** | The acute angles of a right triangle are complementary.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>∠R is a right angle.</td>
<td>Given</td>
</tr>
<tr>
<td>( m \angle R + m \angle S + m \angle T = 180 )</td>
<td>Angle sum theorem</td>
</tr>
<tr>
<td>( m \angle R = 90 )</td>
<td>Definition of right angle</td>
</tr>
<tr>
<td>( 90 + m \angle S + m \angle T = 180 )</td>
<td>Substitution property of equality</td>
</tr>
<tr>
<td>( m \angle S + m \angle T = 90 )</td>
<td>Subtraction property of equality</td>
</tr>
<tr>
<td>( \angle S ) and ( \angle T ) are complementary</td>
<td>Definition of complementary angles</td>
</tr>
</tbody>
</table>

24. **Corollary** | There can be at most one right or obtuse angle in a triangle

In \( \triangle MNO \), \( \angle M \) is a right angle. \( m \angle M \) is a right angle.
\( m \angle M + m \angle N + m \angle O = 180 \)
\( m \angle M = 90, \) so \( m \angle N + m \angle O = 90 \)

If \( \angle N \) were a right angle, then \( m \angle O = 0 \). But that is impossible, so there cannot be two right angles in a triangle.

In \( \triangle PQR \), \( \angle P \) is obtuse. So \( m \angle P > 90 \).
\( m \angle P + m \angle Q + m \angle R = 180 \)

It must be that \( m \angle Q + m \angle R < 90 \). So \( \angle Q \) and \( \angle R \) must be acute.
How Do You Measure a Triangle? Worksheet

1. Name some everyday items that are shaped like triangles. Classify each triangle by angles and by sides.

2. Draw an isosceles right triangle and label the hypotenuse and legs.

3. Draw and label a scalene obtuse triangle. Identify the side opposite and the obtuse angle.

4. Draw an equilateral triangle and describe the lengths of the sides.

5. Draw a triangle and label an exterior angle at each vertex.

In figure ACDE, \( \angle E \) and \( \angle ADC \) are right angles and the congruent parts are indicated.

6. Name the right triangle(s).

7. Name the isosceles triangle(s).

8. Which triangle(s) is obtuse?

9. Which triangle(s) is equilateral?

10. Which segment(s) can be called the hypotenuse?

11. Which segment(s) is opposite \( \angle C \)?

Find the measure of each angle in the figure at the right.

12. \( m \angle 1 \)

13. \( m \angle 2 \)

14. \( m \angle 3 \)

15. \( m \angle 4 \)

16. \( m \angle 5 \)
Find the value of $x$

17. \[ \triangle ABC \] with $\angle 31^\circ$ and $\angle 42^\circ$.

18. \[ \triangle DEF \] with $\angle 78^\circ$.

19. \[ \triangle GHI \] with $\angle 36^\circ$.

20. Name the exterior angles and their corresponding remote interior angles in the figure at the right.

21. Find the value of $x$.

22. Find the measure of $\angle 6$.

23. Find the measure of $\angle 7$.

24. Find the measure of $\angle 8$.

25. Find the measure of $\angle 9$. 
How Do You Measure a Triangle? Worksheet Key

1. Name some everyday items that are shaped like triangles. Classify each triangle by angles and by sides.

   Answers will vary. A sample answer is a boy’s bicycle frame. It is scalene and acute.

2. Draw an isosceles right triangle and label the hypotenuse and legs.

   See students’ work.

3. Draw and label a scalene obtuse triangle. Identify the side opposite and the obtuse angle.

   See students’ work.

4. Draw an equilateral triangle and describe the lengths of the sides.

   See students’ work.

5. Draw a triangle and label an exterior angle at each vertex.

   See students’ work.

In figure ACDE, \( \angle E \) and \( \angle ADC \) are right angles and the congruent parts are indicated.

6. Name the right triangle(s).

   \( \triangle ADE, \triangle ADC \)

7. Name the isosceles triangle(s).

   \( \triangle BCD, \triangle ABD \)

8. Which triangle(s) is obtuse?

   \( \triangle ABD \)

9. Which triangle(s) is equilateral?

   \( \triangle BCD \)

10. Which segment(s) can be called the hypotenuse?

    \( \overline{AD}, \overline{AC} \)
11. Which segment(s) is opposite $\angle C$?

$\overline{BD}, \overline{AD}$

Find the measure of each angle in the figure at the right.

12. $m \angle 1 \rightarrow 42^\circ$
13. $m \angle 2 \rightarrow 73^\circ$
14. $m \angle 3 \rightarrow 73^\circ$
15. $m \angle 4 \rightarrow 65^\circ$
16. $m \angle 5 \rightarrow 65^\circ$

Find the value of $x$

17. $x = 107^\circ$
18. $2x = 102$
$x = 51^\circ$

19. $x = 90 + 35$
$x = 125^\circ$

20. Name the exterior angles and their corresponding remote interior angles in the figure at the right.

$\angle 1: \angle 3, \angle 4$
$\angle 5: \angle 3, \angle 4$
$\angle 6: \angle 2, \angle 4$
$\angle 7: \angle 2, \angle 4$
$\angle 8: \angle 2, \angle 3$
$\angle 9: \angle 2, \angle 3$
21. Find the value of x.

\[(3x + 16) + (45) + (68) = 180\]
\[3x + 129 = 180\]
\[3x = 51\]
\[x = 17\]

22. Find the measure of \(\angle 6\).

\[35 + \angle 6 = 90 \Rightarrow \angle 6 = 55^\circ\]

23. Find the measure of \(\angle 7\).

\[\angle 6 = \angle 7 \Rightarrow \angle 7 = 55^\circ\]

24. Find the measure of \(\angle 8\).

\[\angle 8 = 90^\circ \Rightarrow \text{alternate interior angles}\]

25. Find the measure of \(\angle 9\).

\[\angle 7 + \angle 8 + \angle 9 = 180\]
\[55 + 90 + \angle 9 = 180\]
\[\angle 9 = 35^\circ\]
How Do You Measure a Triangle? Checklist

1. On question 1, did the student answer question correctly?
   a. Yes (10 points)
   b. Named everyday items but did not classify correctly (5 points)

2. On questions 6 thru 11, did the student answer questions correctly?
   a. Yes (30 points)
   b. 5 of the 6 (25 points)
   c. 4 of the 6 (20 points)
   d. 3 of the 6 (15 points)
   e. 2 of the 6 (10 points)
   f. 1 of the 6 (5 points)

3. On questions 12 thru 16, did the student name the angle correctly?
   a. Yes (25 points)
   b. 4 of the 5 (20 points)
   c. 3 of the 5 (15 points)
   d. 2 of the 5 (10 points)
   e. 1 of the 5 (5 points)

4. On questions 17 thru 19, did the student find x correctly?
   a. Yes (15 points)
   b. 2 of the 3 (10 points)
   c. 1 of the 3 (5 points)

5. On question 20, did the student answer the question correctly?
   a. Yes (10 points)
   b. Named the exterior angles but not the corresponding remote interior angles (5 points)

6. On question 21, did the student find the value of x correctly?
   a. Yes (5 points)

7. On questions 22 thru 25, did the student find the value of the angles correctly?
   a. Yes (20 points)
   b. 3 of the 4 (15 points)
   c. 2 of the 4 (10 points)
   d. 1 of the 4 (5 points)

Total Number of Points __________
1. Does the student need remediation in content (labeling and classifying triangles) for questions 1 thru 5? Yes_______  No_______

2. Does the student need remediation in content (identifying specific triangles) for questions 6 thru 11? Yes_______  No_______

3. Does the student need remediation in content (finding the measure of angles involving parallel lines) for questions 12 thru 16? Yes_______  No_______

4. Does the student need remediation in content (naming exterior and remote interior angles) for question 20? Yes_______  No_______

5. Does the student need remediation in content (solving equations involving degree measures of a triangle) for question 21? Yes_______  No_______

6. Does the student need remediation in content (identifying congruent angles in a triangle) for questions 22 thru 25? Yes_______  No_______

A  108 points and above
B  103 points and above
C  92 points and above
D  81 points and above
F  80 points and below

NOTE: The sole purpose of this checklist is to aide the teacher in identifying students that need remediation. It is suggested that teacher’s devise their own point range for determining grades. In addition, some students need remediation in specific areas. The following checklist provides a means for the teacher to assess which areas need addressing.