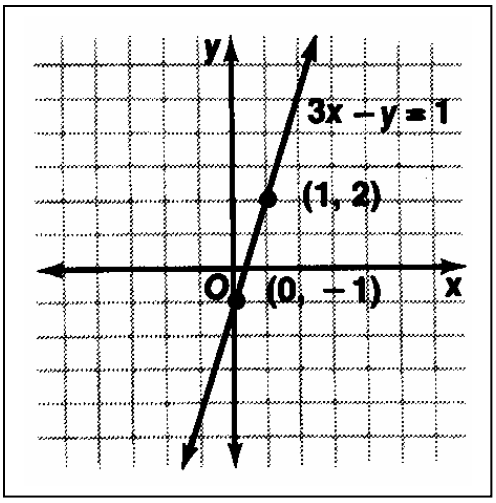
Sloping and Intersecting a Linear Function Examples

1. You can write the solutions to open sentences in two variables as sets of ordered pairs. These solutions can be graphed in the coordinate plane. The following graph represents the solutions to 3x – y = 1.



Make sure that students understand that 3x – y = 1 means 3 times a number minus a second number equals 1.

Have students state some ordered pairs that meet the requirements of the function. Some examples include: (0, –1), (1, 2), (2, 5), (–1, –4), (–2, –7).

When variables other than x and y are used, assume that the letter coming first in the alphabet represents the domain or horizontal coordinate.

2. An infinite number of ordered pairs will satisfy 3x – y = 1. The graph of these ordered pairs is a straight line. An equation whose graph is a straight line is called a **linear equation**.

3. In a linear equation, each term is a constant, like 7, or a constant times a variable to the first power, like 3x. Thus, 4x + 3y = 7, and y = 8, 5m – n = 1, and y = 7 + 2x are linear equations. But $3x + y^2 = y$ and $\frac{1}{x} + y$ are not linear equations? Why

The equations are not linear because they contain variables that are not to the first power.

4. Any linear equation can be written in **standard form**.

Standard Form of a Linear Equation	**Ax + By = C** **Where A, B, and C are real numbers, and A and B are not both zero.**

5. **Example** – Write the equation $x = \frac{2}{3}y - 1$ in standard form.

$x = \frac{2}{3}y - 1$ ➔ 3x = 2y – 3 ➔ 3x – 2y = –3

Multiply each side by 3 to eliminate the fraction.
Add –2y to each side.

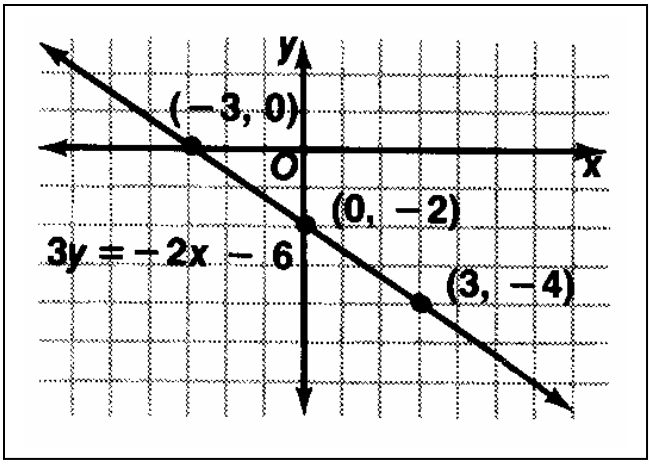
6. To graph a linear equation, it is helpful to make a table of ordered pairs that satisfy the equation. These ordered pairs can then be graphed and connected in a straight line. Since two points determine a line, you need only two points to graph a linear equation in two variables. In checking your work, it is helpful to use a third point.

7. **Example** – Graph $3y = -2x - 6$.

First, solve the equation for y.

$3x = -2x - 6$.

$y = -\frac{2}{3}x - 2$



Next, find three ordered pairs that satisfy the equation. Then, graph the ordered pairs and connect the points with a line.

x	$-\frac{2}{3}x - 2$	y	(x, y)
–3	$-\frac{2}{3}(-3) - 2$	0	(–3, 0)
0	$-\frac{2}{3}(0) - 2$	–2	(0, –2)
3	$-\frac{2}{3}(3) - 2$	–4	(3, –4)

Point out that students may want to use 0 as one x-value, alone with one negative number and one positive number. In this example, the values –3 and 3 were chosen because they can be multiplied easily by $\frac{2}{3}$.

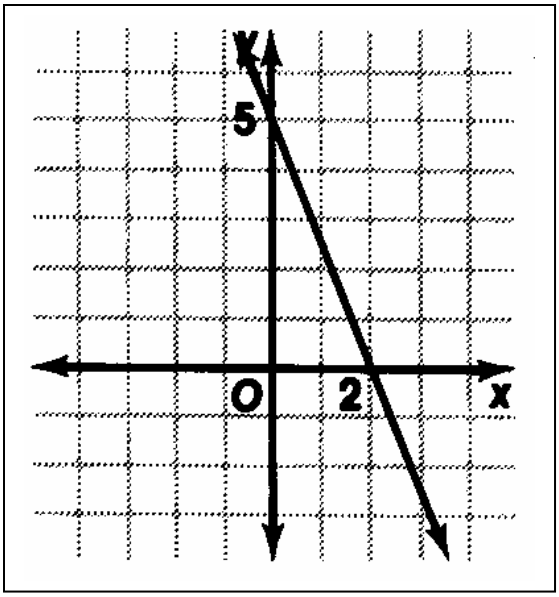
8. **Example** – Graph $5x + 2y = 10$.

Solve $5x + 2y = 10$ for y.

$2y = -5x + 10$

$y = -\frac{5}{2}x + 5$

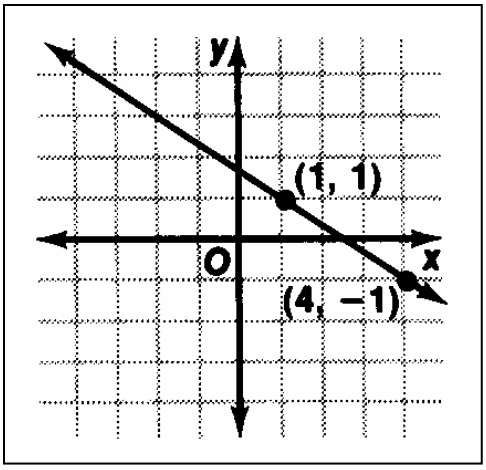
x	$-\frac{5}{2}x + 5$	y	(x, y)
–2	$-\frac{5}{2}(-2) + 5$	10	(–2, 10)
0	$-\frac{5}{2}(0) + 5$	5	(0, 5)
2	$-\frac{5}{2}(2) + 5$	0	(2, 0)



9. **Example** – Graph 2x = 5 – 3y.

Solve 2x = 5 – 3y for y.
3y = –2x + 5
$y = -\frac{2}{3}x + \frac{5}{3}$

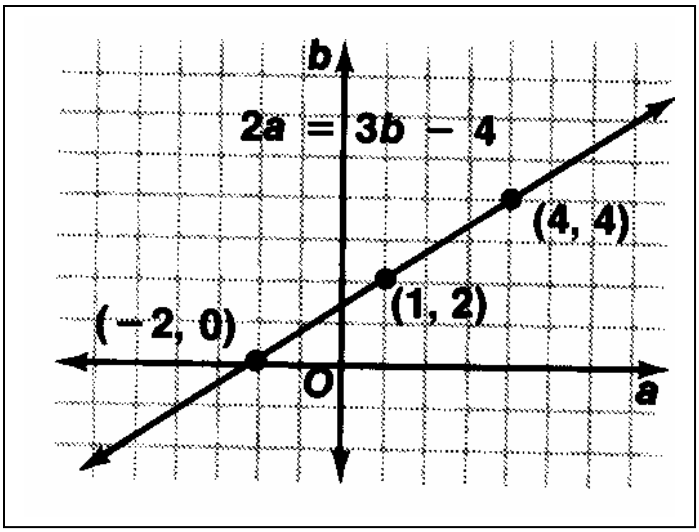
x	$-\frac{2}{3}x + \frac{5}{3}$	y	(x, y)
1	$-\frac{2}{3}(1) + \frac{5}{3}$	1	(1, 1)
0	$-\frac{2}{3}(0) + \frac{5}{3}$	$\frac{5}{3}$	$(0, \frac{5}{3})$
4	$-\frac{2}{3}(4) + \frac{5}{3}$	–1	(4, –1)



10. **Example** – Graph 2a = 3b **– 4.**

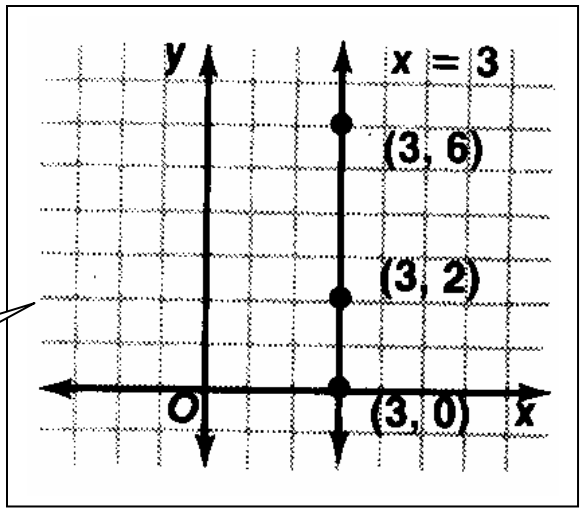
Solve 2a = 3b – 4 for b.
–3b = –2a – 4
$b = \frac{2}{3}a + \frac{4}{3}$

a	$\frac{2}{3}a + \frac{4}{3}$	b	(a, b)
–2	$\frac{2}{3}(-2) + \frac{4}{3}$	0	(–2, 0)
1	$\frac{2}{3}(1) + \frac{4}{3}$	1	(1, 1))
4	$\frac{2}{3}(4) + \frac{4}{3}$	4	(4, 4)



11. Not all linear equations represent functions. For example, consider the equation x = 3. Its graph is a vertical line. Any vertical line drawn on the graph of the equation passes through every point of that graph. The relation is not a function.

The equation x = 3 means y can have any value as long as x is 3.



12. The linear equations y = c and x = c represent two special cases of **straight lines**.

The graph of y = c is a horizontal line passing through (0, c). This is called the constant function.

The graph of x = c is a vertical line passing through (c, 0). x =c does not represent a linear function; there are an infinite number of values paired with c such as (c, 1), (c, 2), (c, 3) and so on.

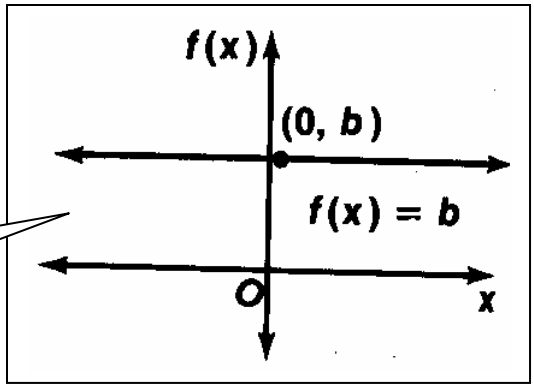
13.

Definition of Linear Function	**A function is linear if it can be defined by f(x) = mx + b where m and b are real numbers.**

Note the similarity between the equation of a line and the notation for a linear function. The linear equation y = 4x + 7 becomes the function f(x) = 4x + 7. Consider an equation in standard form. The equation 3x – y = 1 is a linear equation and is equivalent to y = 3x – 1 when solving for y. Thus 3x – y = 1 can be expressed as the function f(x) = 3x – 1.

14. In the definition of a linear function, m or b may be zero. If m = 0, then f(x) = b. The graph is a horizontal line. This function is called a **constant function**.

f(x) = 0 is called the zero function.



15. **Example** – Is $f(x) = x^3 + 4$ a linear function?

This cannot be written in the form f(x) = mx + b. Thus, it is not a linear function.

16. **Example** – Is g(x) = 4 – x a linear function?

This can be written in the form of f(x) = mx + b where m = –1 and b = 4. Thus it is a linear function.

17. **Example** – Is $f(x) = 3(x^2 - 1) + x - 3x^2$ a linear function?

Yes, f(x) = x – 3, m = 1, b = –3

18. **Example** – Is g(x) = 3x + xy – 4 a linear function?

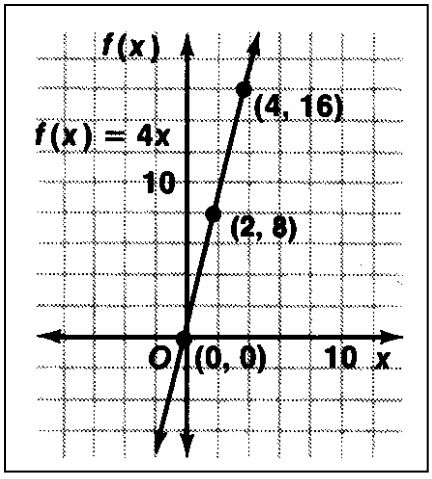
This cannot be written in the form g(x) = mx + b. Thus, it is not a linear function.

19. A ramp was installed to give skate boarders a place to practice. The base is 12 meters long and an elevation of 2 meters. The steepness or slope of the ramp is found by using the following ratio.

Slope = $\dfrac{slope\_in\_vertical\_units}{change\_in\_horizontal\_units}$

Thus, the slope of the ramp is $\frac{2}{12}$ **or** $\frac{1}{6}$**.**

20. Slope is also defined for the graphs of linear functions. In the graph of f(x) = 4x shown at the right, the y-coordinates increase 8 units for each 2 units increase in the corresponding x-coordinates. The slope of the line whose equation is f(x) = 4x is $\frac{8}{2}$ or 4. The vertical change is the difference of the y-coordinates. The horizontal change is the difference of the corresponding x-coordinates.



	x	4x	
+2	0	0	+8
	2	8	
	4	16	

21.

Definition of Slope	**The slope, m, of a line passing through points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$.**

When discussing the slope formula, emphasize that any two points may be used to compute the slope. Students should write the coordinates of points in corresponding order. Note that $\frac{y_2 - y_1}{x_2 - x_1}$ and $\frac{y_1 - y_2}{x_1 - x_2}$ both produce the correct values for slope. However, $\frac{y_2 - y_1}{x_1 - x_2}$ produces the opposite value.

22. **Example** – Determine the slope of the line that passes through (1, –3) and (0, –5).

$$\mathbf{m} = \frac{y_2 - y_1}{x_2 - x_1}$$

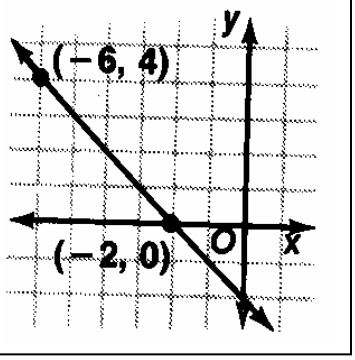
$$\mathbf{m} = \frac{-5-(-3)}{0-1}$$

$$\mathbf{m} = \frac{-2}{-1}$$

$$\mathbf{m = 2}$$

The slope of the line is 2.

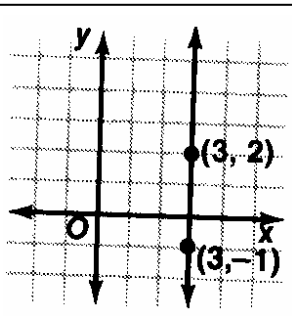
23. **Example** – Determine the slope of the graphs below.



$$\mathbf{m} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\mathbf{m} = \frac{4-0}{-6-(-2)}$$

$$\mathbf{m} = \frac{4}{-4} \rightarrow \mathbf{m = -1}$$

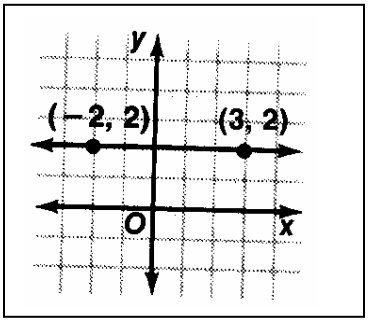


$$\mathbf{m} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\mathbf{m} = \frac{2-(-1)}{3-3}$$

$$\mathbf{m} = \frac{3}{0} \rightarrow$$

m = undefined



$$\mathbf{m} = \frac{y_2 - y_1}{x_2 - x_1}$$

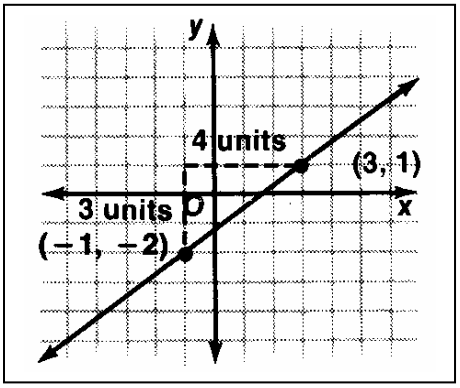
$$\mathbf{m} = \frac{2-2}{-2-3}$$

$$\mathbf{m} = \frac{0}{-5} \rightarrow \mathbf{0}$$

24. Graph the line that passes through (–1, –2) and whose slope $\frac{3}{4}$.

First graph the ordered pair (–1, –2) and whose slope is $\frac{3}{4}$, the vertical change is 3 and the horizontal change is 4. From (–1, –2) move 3 units up and 4 units to the right. This point is (3, 1).

Connect these two points with a straight line.



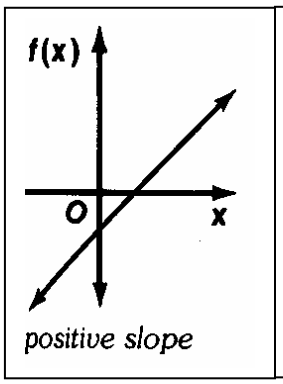
25. The slope of a line may occur in one of the four following ways.

If the line rises to the right, then the slope is positive.

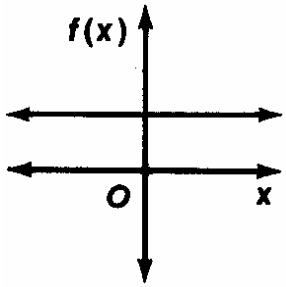
If the line is horizontal, then the slope is zero.

If the line falls to the right, then the slope is negative.

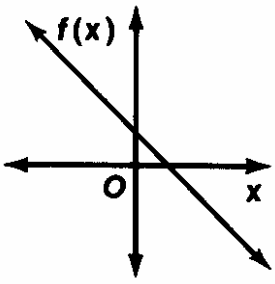
If the is vertical, then the slope is undefined.



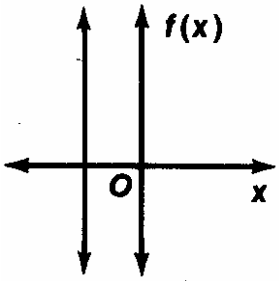
positive slope



zero slope



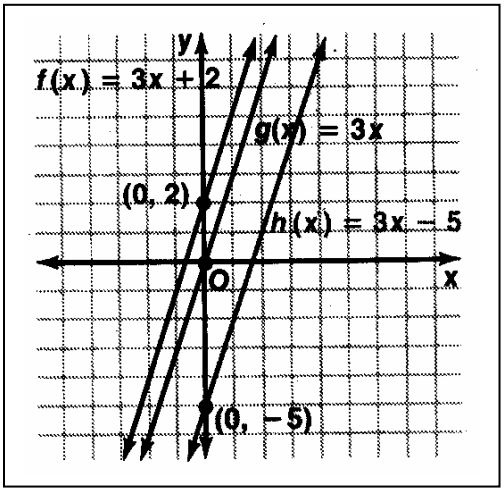
negative slope



undefined slope

A vertical line is also said to have no slope. Students sometimes confuse no slope with zero slope, although they do not mean the same.

26. The graphs of f(x) = 3x + 2, g(x) = 3x, and h(x) = 3x – 5 are lines with the same slope. But, these lines do not pass through the same point.



The slope of each line is 3. Note that 3 is also the coefficient of x in each equation. In general, the slope of a line is the coefficient of x when its equation is solved for y.

27. In the above graph, consider the points where each line crosses the y-axis.

f(x) = 3x + 2 crosses at (0, 2)
g(x) = 3x crosses at (0, 0)
h(x) = 3x – 5 crosses at (0, –5)

The x-coordinate of each point is 0.

The y-coordinates of these points are called the y-intercepts of the lines. The y-intercept is the value of y when x is zero.

28. The x-intercept of a line is the value of x and y is zero. What are the x-intercepts of the lines described above?

f(x) = 3x + 2 has x-intercept of $-\frac{2}{3}$.
g(x) = 3x has x-intercept 0.
h(x) = 3x – 5 has x-intercept of $\frac{5}{3}$.

29. **Example** – Find the y–intercept and x–intercept of the line whose equation is 5x + 3y = 9.

To find the y–intercept, let x = 0 and solve for y.

5(0) + 3y = 9
3y = 9
y = 3 ➔The y–intercept is 3.

To find the x–intercept, let y = 0 and solve for x.

5x + 3(0) = 9
5x = 9
x = $\frac{9}{5}$ ➔The y–intercept is $\frac{9}{5}$.

30. **Example** – Find the missing coordinate if the line passing through the two points (x, 3), and (4, 6) with a slope of –3.

$$\mathbf{m} = \frac{x_2 - x_1}{y_2 - y_1}$$

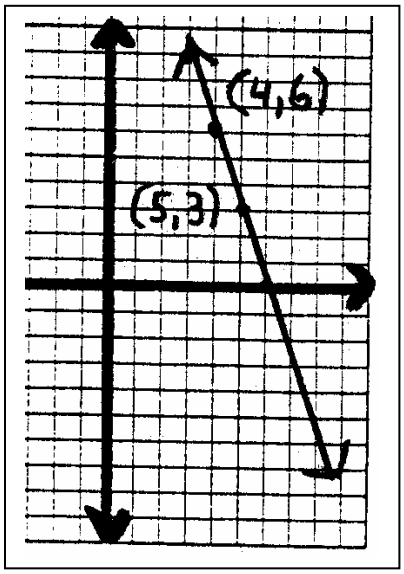
$$\frac{-3}{1} = \frac{6-3}{4-x}$$

3 = –3(4 – x)

3 = –12 + 3x

5 = x

Use a graph to illustrate the x-intercept of the line 5x + 3y = 9



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Sloping and Intersecting a linear Function Worksheet

State whether each equation is a linear equation.

1. $x^2 + y^2 = 7$
2. $x + y = 4$
3. $x – 2y = 5$
4. $x^2 = 9$
5. $a + 3b = 7$
6. $5m^2 = n^2$
7. $y = –4x$
8. $7 = 2y$

State whether each of the following is a linear function.

9. $f(x) = x^2 + 3$
10. $g(x) = 7$
11. $3[g(x)] = x$
12. $g(x) = x – 4$

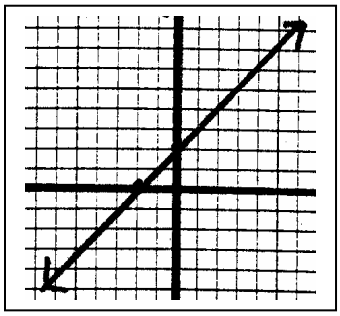
Write each equation in standard form.

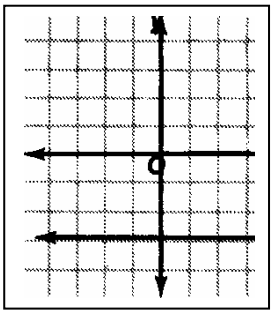
13. $y = 2x – 6$
14. $y = \frac{5}{8}x + 1$

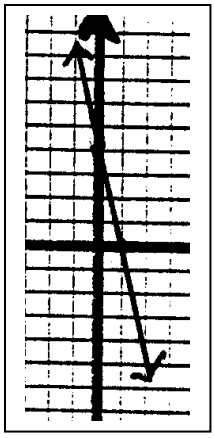
Graph each equation by using a table (provide students with graph paper).

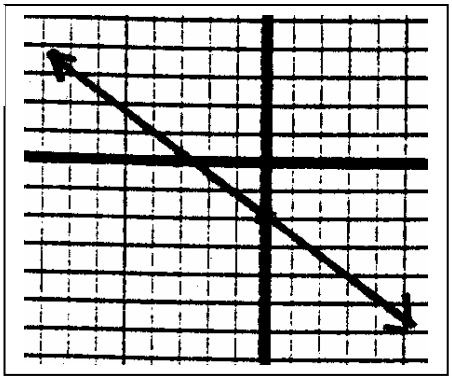
15. $y = x$
16. $y = x + 1$
17. $x + y = 7$
18. $f(x) = 2x + 1$
19. $5x = 10$
20. $3y = 9$

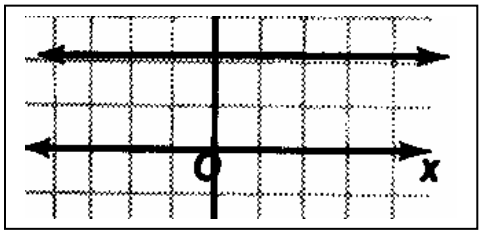
State the slope, y-intercept, and x-intercept of each line

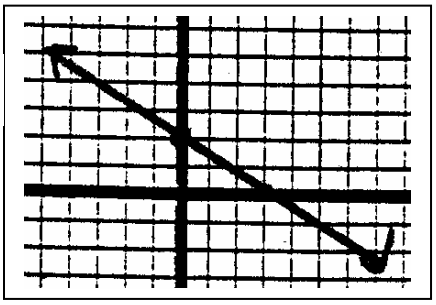
21. 

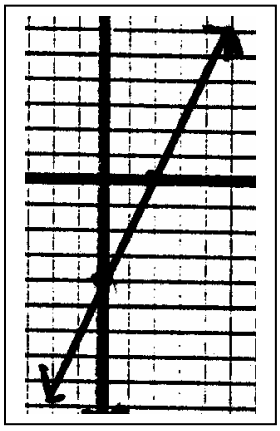
22. 

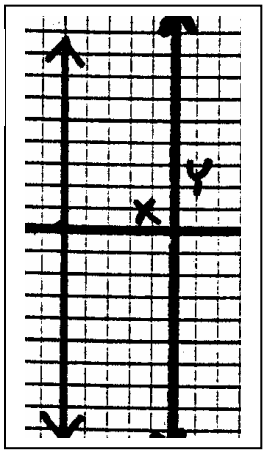
23. 

24. 

25. 

26. 

27. 

28. 

Determine the slope of the line passing through each pair of points

29. (6, 1) and (8, –4)

30. (–3, 0) and (8, 2)

31. (6, 1) and (6, 7)

32. $(1\frac{3}{4}, \frac{1}{3})$ and $(2, \frac{1}{3})$

Find the y-intercept and x-intercept of each line.

33. $y = 5x - 9$

34. $y - 1 = 7x$

35. $3y = x + 4$

36. $y = -2$

37. $x = 4$

38. $f(x) = x - 2$

39. $2x + 3y = 6$

Graph the line that passes through the given point and has the given slope

40. (–3, 2), m = –2

41. (0, 0), m = 3

42. (–1, 1), $m = \frac{1}{4}$

43. (2, –1), m = 0

44. (–3, –1), undefined

Find the missing coordinate if the line passing through the two points has the given slope.

45. (0, 0) and (x, 7), m = 7

46. (6, y) and (2, –13), m = 3

Sloping and Intersecting a Linear Function Worksheet Key

State whether each equation is a linear equation.

1. $x^2 + y^2 = 7$ ➔**No**
2. $x + y = 4$ ➔**Yes**
3. $x - 2y = 5$ ➔**Yes**
4. $x^2 = 9$ ➔**No**
5. $a + 3b = 7$ ➔**Yes**
6. $5m^2 = n^2$ ➔**No**
7. $y = -4x$ ➔**Yes**
8. $7 = 2y$ ➔ **Yes**

State whether each of the following is a linear function.

9. $f(x) = x^2 + 3$ ➔ **No**
10. $g(x) = 7$ ➔ **Yes**
11. $3[g(x)] = x$ ➔ **Yes**
12. $g(x) = x - 4$ ➔ **Yes**

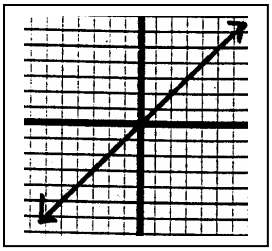
Write each equation in standard form.

13. $y = 2x - 6$ ➔**2x – y = 6**
14. $y = \frac{5}{8}x + 1$➔ **5x – 8y = –8**

Graph each equation (provide students with graph paper).

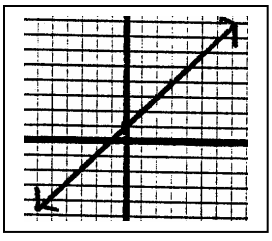
15. $y = x$

x	y
0	0
1	1
–1	–1
2	2
–2	–2



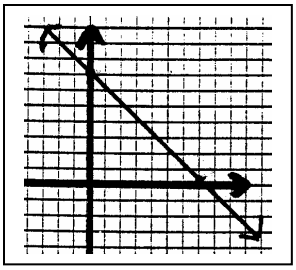
16. $y = x + 1$

x	y
0	1
1	2
–1	0
2	3
–2	–1



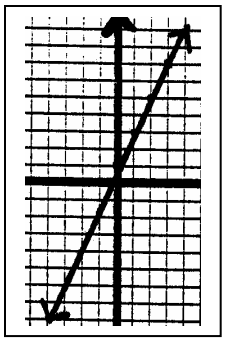
17. $x + y = 7$

x	y
0	7
1	6
–1	8
2	5
–2	9



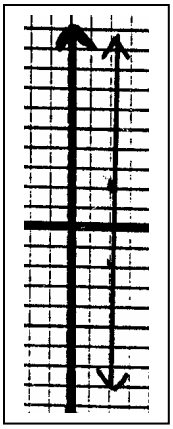
18. f(x) = 2x + 1

x	y
0	1
1	3
–1	–1
2	5
–2	–3



19. 5x = 10

x = 2

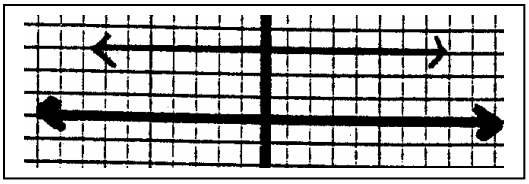


x	y
2	0
2	1
2	–1
2	2
2	–2

20. 3y = 9

y = 3

x	y
0	3
1	3
–1	3
2	3
–2	3



State the slope, y-intercept, and x-intercept of each line

21. **Slope = 1 (right 1, up 1), y-intercept (0, 2), x-intercept (–2, 0)**

22. **Slope = 0 (horizontal line), y-intercept (0, –3), x-intercept (none)**

23. **Slope = –4 (right 1, down 4), y-intercept (0, 4), x-intercept (1, 0)**

24. **Slope =** $-\frac{2}{3}$ **(right 3, down 2), y-intercept (0, –2), x-intercept (–3, 0)**

25. **Slope = 0 (horizontal line), y-intercept (0, 2), x-intercept (none)**

26. **Slope =** $-\frac{2}{3}$ **(right 3, down 2), y-intercept (0, 2), x-intercept (3, 0)**

27. **Slope = 2 (right 1, up 2), y-intercept (0, –4), x-intercept (2, 0)**

28. **Slope = (undefined – vertical line), y-intercept (none), x-intercept (–5, 0)**

Determine the slope of the line passing through each pair of points

29. (6, 1) and (8, –4) $\frac{1-(-4)}{6-8} = -\frac{5}{2}$

30. (–3, 0) and (8, 2) $\frac{0-2}{-3-8} = \frac{2}{11}$

31. (6, 1) and (6, 7) $\frac{1-7}{6-6} = \frac{-6}{0}$ **= undefined**

32. $(1\frac{3}{4}, \frac{1}{3})$ and $(2, \frac{1}{3})$ $\frac{\frac{1}{3}-\frac{1}{3}}{1\frac{3}{4}-2} = 0$

Find the y-intercept and x-intercept of each line.

33. y = 5x – 9

y-intercept (0, -9)

x-intercept ➔0 = 5x – 9 ➔($\frac{9}{5}$,0)

34. y – 1 = 7x

y = 7x + 1 ➔ y-intercept (0, 1)

x-intercept ➔ 0 = 7x + 1 ➔($-\frac{1}{7}$,0)

35. 3y = x + 4

y = $\frac{1}{3}$x + $\frac{4}{3}$ ➔ y-intercept (0, $\frac{4}{3}$)

x-intercept ➔ 0 = $\frac{1}{3}$x + $\frac{4}{3}$ ➔(-4,0)

36. y = –2

y-intercept ➔(0, –2)
x-intercept ➔ (none)

37. x = 4

y-intercept ➔(none)
x-intercept ➔ (4, 0)

38. f(x) = x – 2

y-intercept ➔(0, -2)
x-intercept ➔ (2, 0)

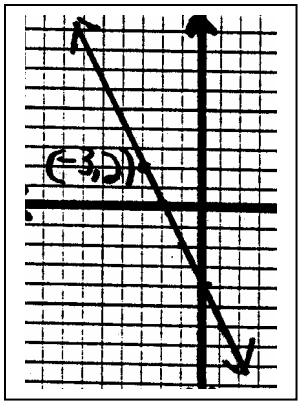
39. 2x + 3y = 6

y = $-\frac{2}{3}$x + 2 ➔ y-intercept (0, 2)

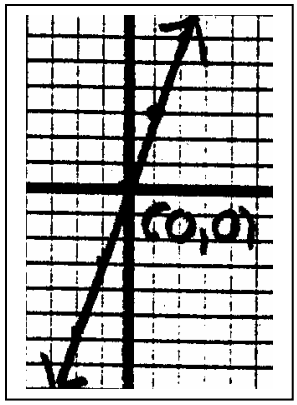
x-intercept ➔ 0 = $-\frac{2}{3}$x + 2 ➔(3, 0)

Graph the line that passes through the given point and has the given slope

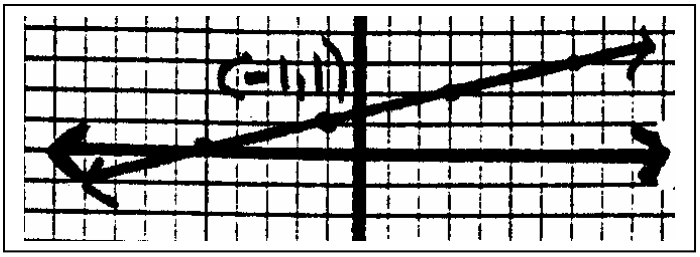
40. (–3, 2), m = –2



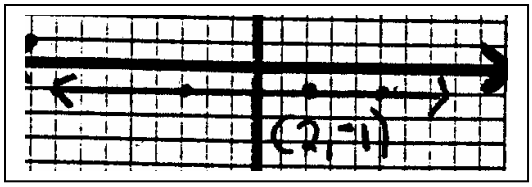
41. (0, 0), m = 3



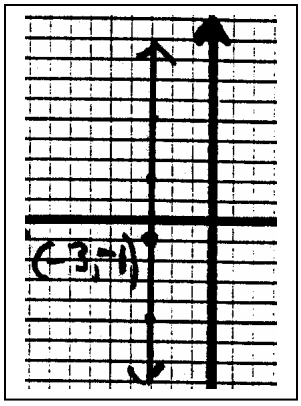
42. (-1, 1), m = $\frac{1}{4}$



43. (2, –1), m = 0



44. (-3, -1), m = undefined



Find the missing coordinate if the line passing through the two points has the given slope.

44. (0, 0) and (x, 7), m = 7

$$\mathbf{7} = \frac{7-0}{x-0}$$

$$\mathbf{7} = \frac{7}{x}$$

$$\mathbf{x = 1}$$

46. (6, y) and (2, –13), m = 3

$$\mathbf{3} = \frac{y-(-13)}{6-2}$$

$$\mathbf{3} = \frac{y+13}{4} \rightarrow \mathbf{12 = y + 13}$$

$$\mathbf{-1 = y}$$

Student Name: _____

Date: _____

Sloping and Intersecting a Linear Function Checklist

1. On questions 1 thru 8, did the student state whether the equation was a linear equation correctly?

   a. Yes (40 points)
   b. 7 out of 8 (35 points)
   c. 6 out of 8 (30 points)
   d. 5 out of 8 (25 points)
   e. 4 out of 8 (20 points)
   f. 3 out of 8 (15 points)
   g. 2 out of 8 (10 points)
   h. 1 out of 8 (5 points)

2. On questions 9 thru 12, did the student state whether the function was a linear function correctly?

   a. Yes (20 points)
   b. 3 out of 4 (15 points)
   c. 2 out of 4 (10 points)
   d. 1 out of 4 (5 points)

3. On questions 13 and 14, did the student write the equation is standard form correctly?

   a. Yes (10 points)
   b. 1 out of 2 (5 points)

4. On questions 15 thru 20, did the student graph the equation correctly?

   a. Yes (30 points)
   b. 5 out of 6 (25 points)
   c. 4 out of 6 (20 points)
   d. 3 out of 6 (15 points)
   e. 2 out of 6 (10 points)
   f. 1 out of 6 (5 points)

5. On questions 21 thru 28, did the student state the slope correctly?

   a. Yes (40 points)
   b. 7 out of 8 (35 points)
   c. 6 out of 8 (30 points)
   d. 5 out of 8 (25 points)
   e. 4 out of 8 (20 points)
   f. 3 out of 8 (15 points)
   g. 2 out of 8 (10 points)
   h. 1 out of 8 (5 points)

6. On questions 21 thru 28, did the student state the y-intercept correctly?

   a. Yes (40 points)
   b. 7 out of 8 (35 points)
   c. 6 out of 8 (30 points)
   d. 5 out of 8 (25 points)
   e. 4 out of 8 (20 points)
   f. 3 out of 8 (15 points)
   g. 2 out of 8 (10 points)
   h. 1 out of 8 (5 points)

7. On questions 21 thru 28, did the student state the x-intercept correctly?

    a. Yes (40 points)
    b. 7 out of 8 (35 points)
    c. 6 out of 8 (30 points)
    d. 5 out of 8 (25 points)
    e. 4 out of 8 (20 points)
    f. 3 out of 8 (15 points)
    g. 2 out of 8 (10 points)
    h. 1 out of 8 (5 points)

8. On questions 29 thru 32, did the student determine the slope of the line correctly?

    a. All (20 points)
    b. 3 out of 4 (15 points)
    c. 2 out of 4 (10 points)
    d. 1 out of 4 (5 points)

9. On questions 33 thru 39, did the student find the y-intercept correctly?

    a. Yes (35 points)
    b. 6 out of 7 (30 points)
    c. 5 out of 7 (25 points)
    d. 4 out of 7 (20 points)
    e. 3 out of 7 (15 points)
    f. 2 out of 7 (10 points)
    g. 1 out of 7 (5 points)

10. On questions 33 thru 39, did the student find the x-intercept correctly?

    a. Yes (35 points)
    b. 6 out of 7 (30 points)
    c. 5 out of 7 (25 points)
    d. 4 out of 7 (20 points)
    e. 3 out of 7 (15 points)
    f. 2 out of 7 (10 points)
    g. 1 out of 7 (5 points)

11. On questions 40 thru 44, did the student graph the line correctly?

    a. Yes (25 points)
    b. 4 out of 5 (20 points)
    c. 3 out of 5 (15 points)
    d. 2 out of 5 (10 points)
    e. 1 out of 5 (5 points)

12. On questions 45 and 46, did the student find the missing coordinate correctly?

    a. Yes (10 points)
    b. 1 out of 2 (5 points)

Total Number of Points _____

Student Name: _____

Date: _____

NOTE: The sole purpose of this checklist is to aide the teacher in identifying students that need remediation. It is suggested that teacher's devise their own point range for determining grades. In addition, some students need remediation in specific areas. The following checklist provides a means for the teacher to assess which areas need addressing.

1. Does the student need remediation in content (identifying linear equations) for questions 1 thru 8? Yes_____ No_____

2. Does the student need remediation in content (identifying linear functions) for questions 9 thru 12? Yes_____ No_____

3. Does the student need remediation in content (writing equations in standard form) for questions 13 and 14? Yes_____ No_____

4. Does the student need remediation in content (graphing linear equations) for questions 15 thru 20? Yes_____ No_____

5. Does the student need remediation in content (stating slope, y-intercept and x-intercept from a graph) for questions 21 thru 28?
   Yes_____ No_____

6. Does the student need remediation in content (determining the slope of a line passing through a pair of points) for questions 29 thru 32?
   Yes_____ No_____

7. Does the student need remediation in content (finding the y-intercept and x-intercept from a linear equation) for questions 33 thru 39?
   Yes_____ No_____

8. Does the student need remediation in content (graphing a line given a point and the slope of the line) for questions 40 thru 44?
   Yes_____ No_____

9. Does the student need remediation in content (finding a missing coordinate given one point and the slope) for questions 45 and 46?
   Yes_____ No_____

A 327 points and above

B 310 points and above

C 293 points and above

D 241 points and above

F 240 points and below

Sample range points!