Solving Systems of Equations Graphically Examples

1. The cost for renting a car from Shady Grady Rent-a-Car is $18 per day plus 30¢ per mile driven. The cost of renting a similar car from EZ-Rider Rental is $20 per day plus 25¢ per mile driven. Cathy needs to rent a car for one day. Should she rent from Shady Grady or EZ-Rider?

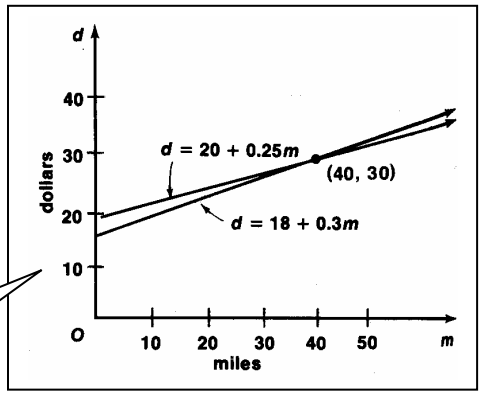
2. **Let c = cost of renting a car for one day.**
   Let m = number of miles driven in one day.

   Using Distance = Rate(Time) you can write the following equations

   c = 18 + 0.30m — **Cost of renting car from Shady Grady.**

   c = 20 + 0.25m — **Cost of renting car from EZ-Rider.**

3. Graphing these two equations shows how the costs compare. The graphs show that the EZ-Rider car costs more if less than 40 miles are driven. EZ-Rider and Shady Grady cost the same if 40 miles are driven. Shady Grady costs more if more than 40 miles are driven.

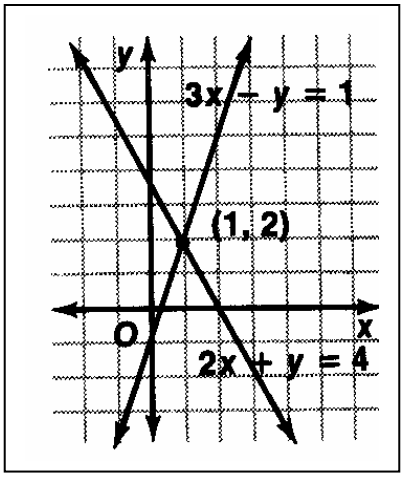


   Each point on a line satisfies the equation of the line. Since (40, 30) is on both lines graphed, it satisfies both equations.

4. Together the equations d = 18 + 0.30m and d = 20 +0.25m are called a **system of equations**. The solution of the system is (40, 30)

   Remind students that the solution, if it exists, of a system with two variables will be an ordered pair or ordered pairs.

5. **Example** – Solve this system by graphing: $3x - y = 1$, $2x + y = 4$

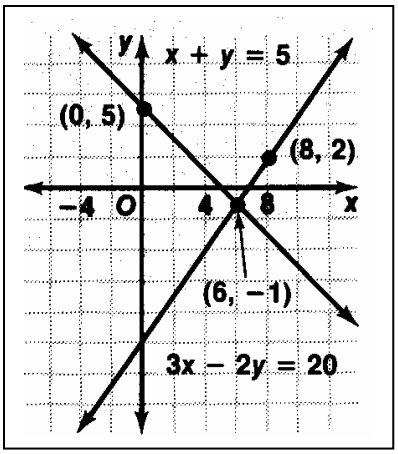


The slope-intercept form of 3x – y = 1 is y = 3x – 1.
The slope-intercept form of 2x + y = 4 is y = –2x + 4.

The two lines have different slopes. The graphs of the equations are intersecting lines.

The solution of the system is (1, 2).

6. **Example** – Solve this system by graphing: $x + y = 5$, $3x - 2y = 20$



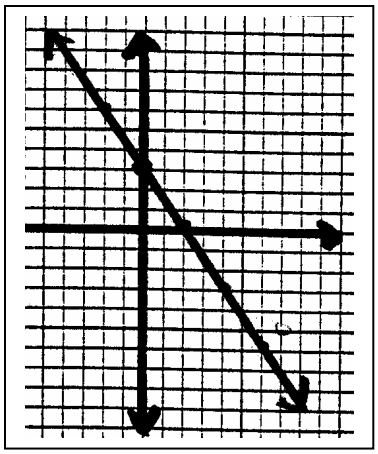
The slope-intercept form of x + y = 5 is y = –x + 5.

The slope-intercept form of 3x – 2y = 20 is y = $\frac{3}{2}$x – 10.

The two lines have different slopes. The graphs of the equations are intersecting lines.

The solution of the system is (6, –1).

7. **Example** – Solve this system by graphing: $2y + 3x = 6$, $4y + 6x = 12$



The slope-intercept form of 2y + 3x = 6 is y = $-\frac{3}{2}$x + 3.

The slope-intercept form of 4y + 6x = 12 is y = $-\frac{3}{2}$x + 3.

Both lines have the same slope and the same y-intercept. The graphs of the equations are the same line. Any ordered pair on the graph satisfies both equations. So, there is an infinite number of solutions to this system of equation.

The solution set is {(x, y)|2y + 3x = 6}

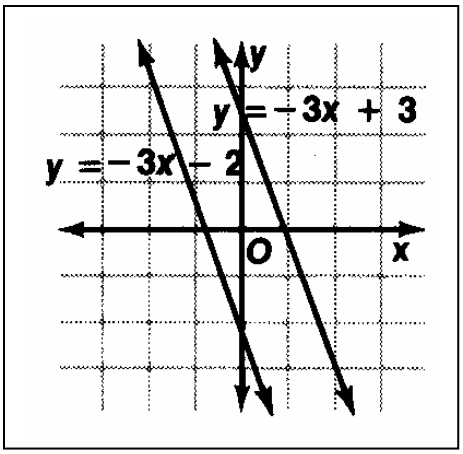
When the graphs of two equations are the same we say the lines coincide.

8. A **consistent** system of equations has at least one solution. For example, the systems of equations in Examples 5, 6, and 7 are consistent. If there is exactly one solution, the system is **independent**. If there is an infinite number of solutions, the system is **dependent**. So, the system in Example 5 and 6 are **consistent** and **independent**. The system in Example 7 is **consistent** and **dependent**.

9. **Example** – Solve this system by graphing: $y = -3x - 2$
$y = -3x + 3$

Both lines have the same slope but different y-intercepts. The graphs of the equations are parallel lines. Since they do not intersect, there are no solutions to this system of equations. Such a system is said to be inconsistent.

The solution set is the empty set, Ø.



10. The following chart gives a summary of the possibilities for the graphs of two linear equations in two variables.

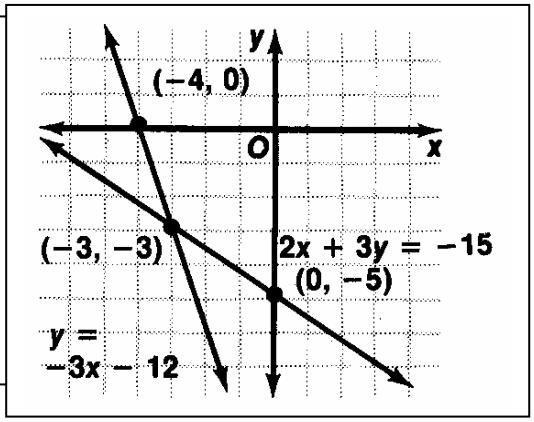
Graphs of equations	Slopes of Lines	Name of System of Equations	Number of Solutions
Lines intersect	Different slopes	Consistent and independent	One
Lines coincide	Same slope, same intercepts	Consistent and dependent	Infinite
Lines parallel	Same slope, different intercepts	Inconsistent	None

11. **Example** – Solve this system by graphing: $y = -3x - 12$
$2x + 3y = -15$

The slope-intercept form of $2x + 3y = -15$ is $y = -\frac{2}{3}x - 5$.

The two lines have different slopes. The graphs of the equations are intersecting lines.

The solution of the system is (–3, –3). The system is consistent and independent.

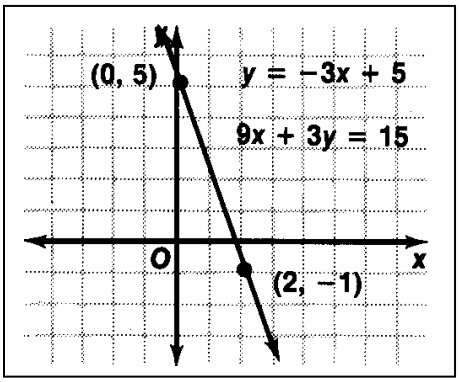


12. **Example** – Solve this system by graphing: $y = -3x + 5$, $9x + 3y = 15$

The slope-intercept form of 9x + 3y = 15 is y = –3x + 5.

Both lines have the same slope and the same y-intercept. The graphs of the equations are the same line. Any ordered pair on the graph satisfies both equations. So, there is an infinite number of solutions to this system of equation.

The solution set is {(x, y)|9x + 3y = 15}. The system is consistent and dependent.

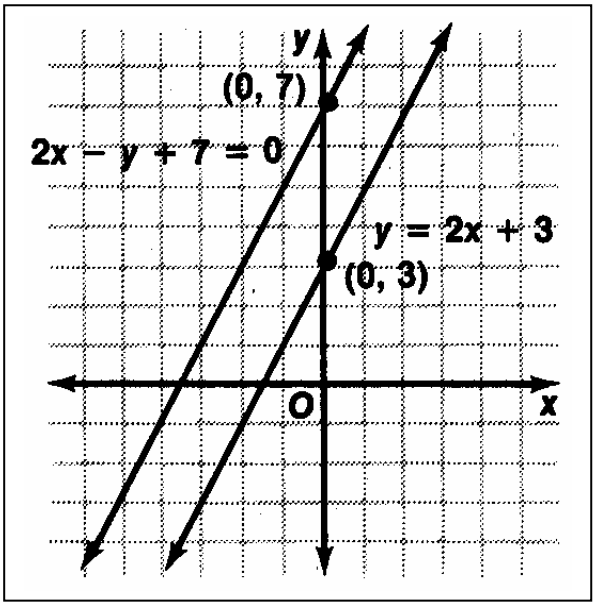


13. **Example** – Solve this system by graphing: $y = 2x + 3$, $2x - y + 7 = 0$

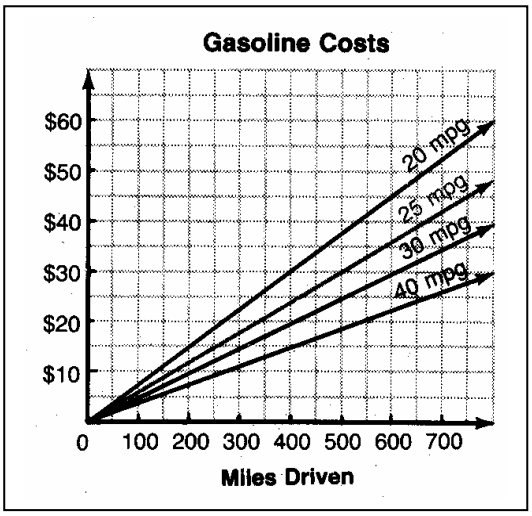
The slope-intercept form of 2x – y + 7 = 0 is y = 2x + 7.

Both lines have the same slope but different y-intercepts. The graphs of the equations are parallel lines. Since they do not intersect, there are no solutions to this system of equations. Such a system is said to be inconsistent.

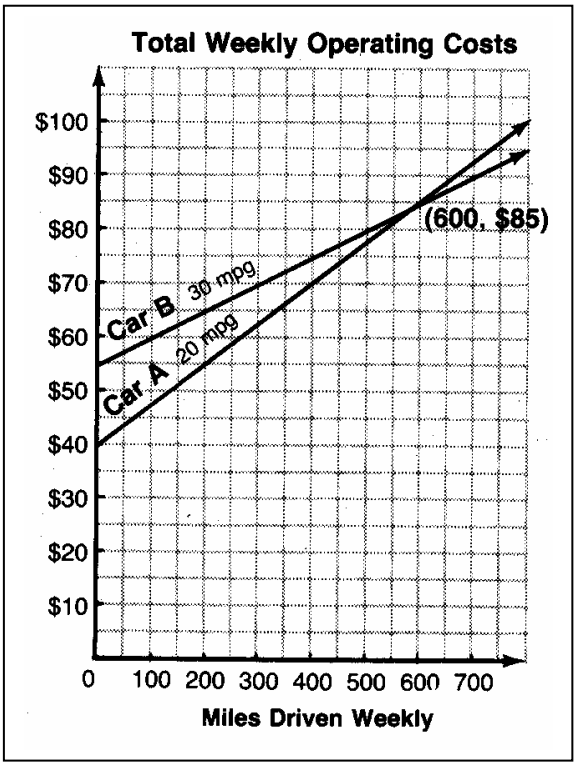
The solution set is the empty set, Ø. The system is inconsistent.



14. The cost of driving a car is affected by the rate at which it uses gasoline. The graph at the right shows the cost of gasoline, based on the number of miles driven and the usage rate in miles per gallon (mpg). (Note: The cost of gasoline for this graph is $1.50 per gallon.)
    Suppose Car A gets an average of 20 miles per gallon and Car B gets 30 miles per gallon. Each car is driven 600 miles. From the graph, you can see that the gasoline cost for Car A is $45 and for Car B is $30.



15. **Example** – There are also fixed expenses for operating a car, such as loan repayment and insurance. The weekly fixed expenses are $40 for Car A, and $55 for Car B. This information can be used to draw a graph that shows the total weekly operating costs for the two cars.
    Each line on the graph at the right has the same slope as the corresponding line on the graph above. However, the intercept of each line is the fixed cost per week.
    Study the graph. The operating cost for Car A is less when a person drives less than 600 miles per week. When is the operating cost for Car B less than for Car A?



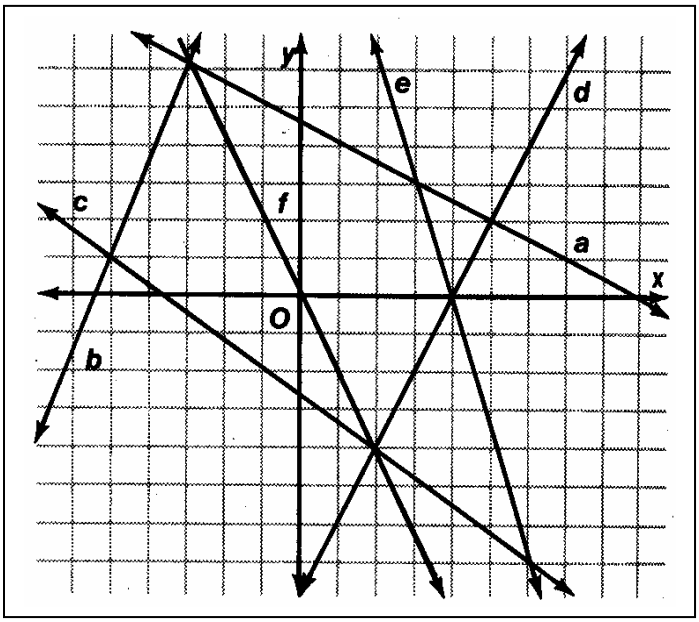
The operating cost for Car B is less than for Car A when a person drives more than 600 miles per week.

Name:_____
Date:_____
Class:_____

Solving Systems of Equations Graphically Worksheet

State the ordered pair that is the intersection of each pair of lines.

1. a, b
2. a, e
3. a, d
4. b, c
5. c, d
6. e, d
7. b, f
8. f, d



Graph each system of equations and state its solution. Then state whether the system is consistent and independent, consistent and dependent, or inconsistent (Give students graph paper).

9. $x + y = 4$
   $2x + 3y = 9$

10. $x + y = 6$
    $x - y = 2$

11. $x + y = 6$
    $3x + 3y = 3$

12. $x + 1 = y$
    $2x - 2y = 8$

13. $\frac{1}{2}x + \frac{1}{3}y = 2$
    $x - y = -1$

14. $x + y = 1$
    $3x + 5y = 7$

15. George bought 7 quarts of cleaning fluid: x quarts at $3.00 per quart and y quarts at $2.00 per quart. Find x and y if the total cost was $16.00.

Name:_____
Date:_____
Class:_____

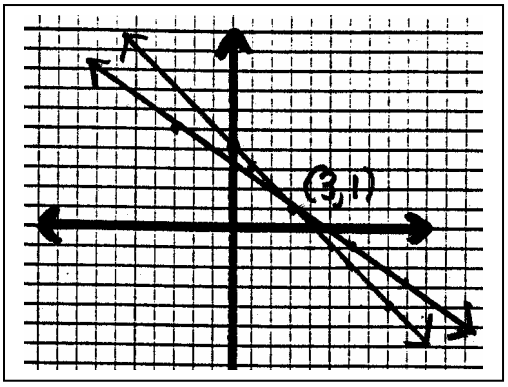
Solving Systems of Equations Graphically Worksheet Key

State the ordered pair that is the intersection of each pair of lines.

1. a, b ➔**(–3, 6)**
2. a, e ➔**(3, 3)**
3. a, d ➔**(5, 2)**
4. b, c ➔**(–5, 1)**
5. c, d ➔**(2, –4)**
6. e, d ➔**(4, 0)**
7. b, f ➔**(–3, 6)**
8. f, d ➔**(2, -4)**

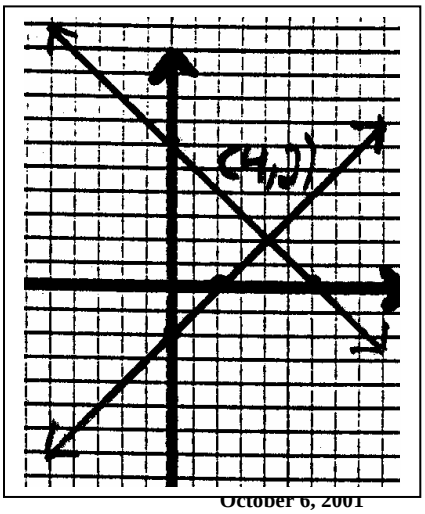
Graph each system of equations and state its solution. Then state whether the system is consistent and independent, consistent and dependent, or inconsistent (Give students graph paper).

9. x + y = 4
   2x + 3y = 9



Slope intercept forms:
y = **–x + 4**
y = $-\frac{2}{3}$x + 3
(3, 1); consistent and independent

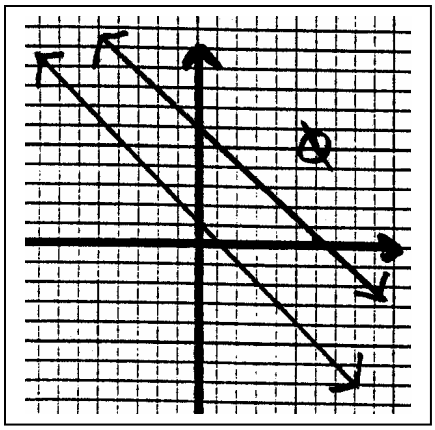
10. x + y = 6
    x – y = 2



Slope intercept forms:
y = **–x + 6**
y = x – 2
(4, 2); consistent and independent

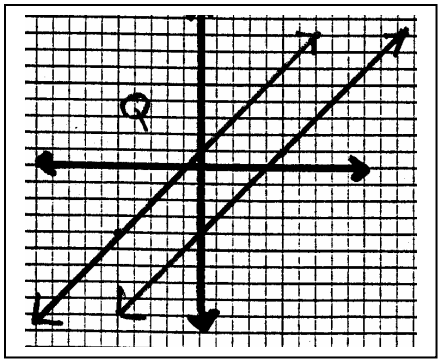
11. $x + y = 6$
$3x + 3y = 3$

Slope intercept forms:
$y = \mathbf{-x + 6}$
$\mathbf{y = -x + 1}$
no solutions; inconsistent



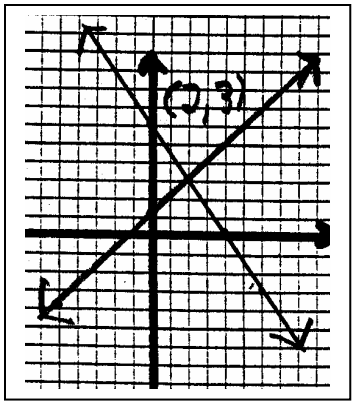
12. $x + 1 = y$
$2x - 2y = 8$

Slope intercept forms:
$y = \mathbf{x + 1}$
$\mathbf{y = x - 4}$
no solutions; inconsistent



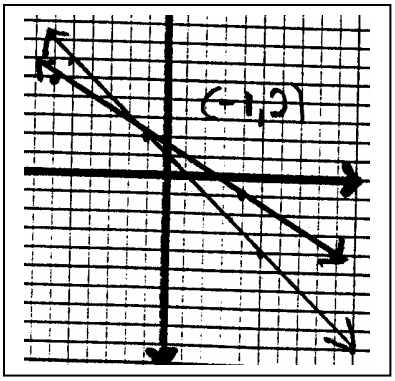
13. $\frac{1}{2}x + \frac{1}{3}y = 2$
$x - y = -1$

Slope intercept forms:
$y = \mathbf{-\frac{3}{2}x + 6}$
$\mathbf{y = x + 1}$
(2, 3); consistent and independent

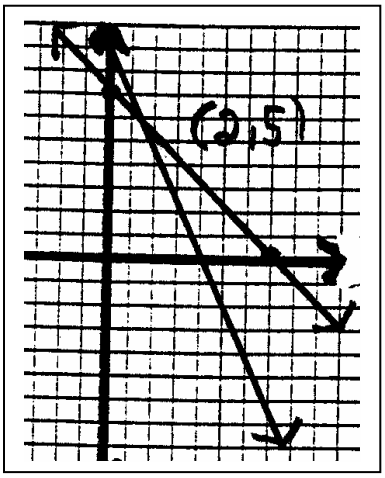


14. $x + y = 1$
$3x + 5y = 7$

Slope intercept forms:
$y = \mathbf{-x + 1}$
$\mathbf{y = -\frac{3}{5}x + \frac{7}{5}}$
(–1, 2); consistent and independent



15. George bought 7 quarts of cleaning fluid: x quarts at $3.00 per quart and y quarts at $2.00 per quart. Find x and y if the total cost was $16.00.



Use x + y = 7 and 3x + 2y = 16 ➔ y = –x + 7 and y = $-\frac{3}{2}$x + 8

Solution is 2 qts at $3 and 5 qts at $2.

Student Name: _____

Date: _____

Solving Systems of Equations Graphically Checklist

1. On questions 1 thru 8, did the student state the correct intersection for each pair of lines?

    a. Yes (40 points)
    b. 7 out of 8 (35 points)
    c. 6 out of 8 (30 points)
    d. 5 out of 8 (25 points)
    e. 4 out of 8 (20 points)
    f. 3 out of 8 (15 points)
    g. 2 out of 8 (10 points)
    h. 1 out of 8 (5 points)

2. On questions 9 thru 14, did the student graph each system correctly?

    a. Yes (30 points)
    b. 5 out of 6 (25 points)
    c. 4 out of 6 (20 points)
    d. 3 out of 6 (15 points)
    e. 2 out of 6 (10 points)
    f. 1 out of 6 (5 points)

3. On questions 9 thru 14, did the student correctly state whether the system is consistent and independent, consistent and dependent, or inconsistent?

    a. Yes (30 points)
    b. 5 out of 6 (25 points)
    c. 4 out of 6 (20 points)
    d. 3 out of 6 (15 points)
    e. 2 out of 6 (10 points)
    f. 1 out of 6 (5 points)

4. On question 7, did the student set up a correct system of equations to solve the problem?

    a. Yes (5 points)

5. On question 7, did the student graph the system of equations correctly?

    a. Yes (5 points)

6. On question 7, did the student find the correct solution?

    a. Yes (5 points)

Total Number of Points _____

Name: _____
Date:_____
Class:_____

NOTE: The sole purpose of this checklist is to aide the teacher in identifying students that need remediation. It is suggested that teacher's devise their own point range for determining grades. In addition, some students need remediation in specific areas. The following checklist provides a means for the teacher to assess which areas need addressing.

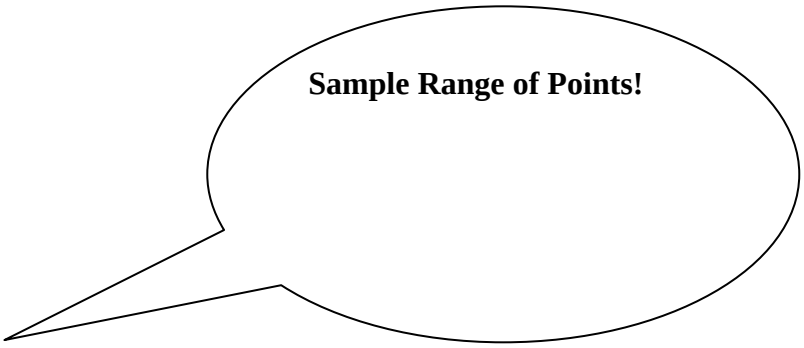
1. Does the student need remediation in content (finding the intersection for linear equations on a graph) for questions 1 thru 8? Yes_____ No_____

2. Does the student need remediation in content (graphing linear functions and find their solution) for questions 9 thru 14?
   Yes_____ No_____

3. Does the student need remediation in content (writing a system of equations from a word problem and solving) for question 15? Yes_____ No_____

4. Does the student need remediation in content (finding the equation of a perpendicular line when given a point and a linear equation) for questions 19 thru 22? Yes_____ No_____

5. Does the student need remediation in content (finding the value of the constant term when given a perpendicular line) for questions 23 thru 26?
   Yes_____ No_____



A 108 points and above

B 97 points and above

C 92 points and above

D 80 points and above

F 79 points and below