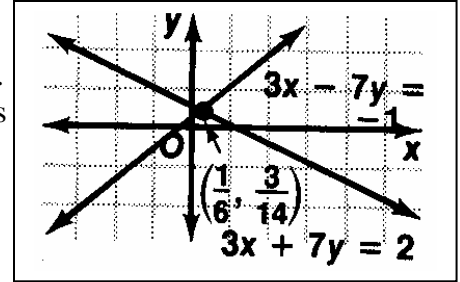


Solving Systems of Equations Algebraically Examples

1. Graphing a system of equations is a good way to determine their solution if the intersection is an integer. However, if the solution is not an integer, the process is not exact.



2. Usually, when a system of equations involves integers and non-integers, it is easier to solve by algebraic methods rather than by graphing. Two such methods are substitution method and the elimination method.
3. Solve the system of equations by substitution:

$$\begin{aligned} 3x - y &= 1 \\ 3x + 2y &= 16 \end{aligned}$$

When solving by substitution, first solve each equation for either y or x (make sure you solve each for the same variable).

$$\begin{aligned} 3x - y &= 1 \rightarrow y = 3x - 1 \\ 3x + 2y &= 16 \rightarrow y = -\frac{3}{2}x + 8 \end{aligned}$$

The solution is $(2, 5)$

Since $y = 3x - 1$ and $y = -\frac{3}{2}x + 8$, then

$$\begin{aligned} 3x - 1 &= -\frac{3}{2}x + 8 \\ \frac{9}{2}x &= 9 \rightarrow x = 2 \end{aligned}$$

To find y , substitute $x = 2$ back into any of the equations.

$$y = 3(2) - 1 \rightarrow y = 5$$

Note: Make sure students understand that you can also solve for x first and reach the same solution.

$$\begin{aligned} 3x - y &= 1 \rightarrow x = \frac{1}{3}y + \frac{1}{3} \\ 3x + 2y &= 16 \rightarrow x = -\frac{2}{3}y + \frac{16}{3} \end{aligned}$$

The solution is $(2, 5)$

$$\begin{aligned} \frac{1}{3}y + \frac{1}{3} &= -\frac{2}{3}y + \frac{16}{3} \\ y &= 5 \end{aligned}$$

$$\begin{aligned} 3x - (5) &= 1 \\ 3x &= 6 \\ x &= 2 \end{aligned}$$

4. Solve the system of equations by substitution: $x + 2y = 5$
 $3x + 5y = 14$

When solving by substitution, first solve each equation for either y or x (make sure you solve each for the same variable).

$$x + 2y = 5 \rightarrow y = -\frac{1}{2}x + \frac{5}{2}$$

$$3x + 5y = 14 \rightarrow y = -\frac{3}{5}x + \frac{14}{5}$$

Since $y = -\frac{1}{2}x + \frac{5}{2}$ and $y = -\frac{3}{5}x + \frac{14}{5}$, then

$$-\frac{1}{2}x + \frac{5}{2} = -\frac{3}{5}x + \frac{14}{5}$$

$$\frac{1}{10}x = \frac{3}{10} \rightarrow x = 3$$

To find y, substitute $x = 2$ back into any of the equations.

$$y = -\frac{1}{2}(3) + \frac{5}{2} \rightarrow y = 1$$

The solution is (3, 1)

5. Solve the system of equations by elimination: $4x + 2y = -8$
 $x - 2y = -7$

Use elimination when you can add the two equations together and eliminate a variable.

$$\begin{array}{r} 4x + 2y = -8 \\ x - 2y = -7 \\ \hline 5x = -15 \\ x = -3 \end{array}$$

Substitute -3 in for x in any equation.

$$4(-3) + 2y = -8 \rightarrow 2y = 4 \\ y = 2$$

The solution is (-3, 2).

6. Make sure that students understand that the substitution method is chosen when one of the equations can be easily solved for none of the variables. The elimination method is chosen when one of the two variables can be easily eliminated by adding the equations.

7. When working with equations, you can multiply or divide each side of an equation by the same number and not change the results.

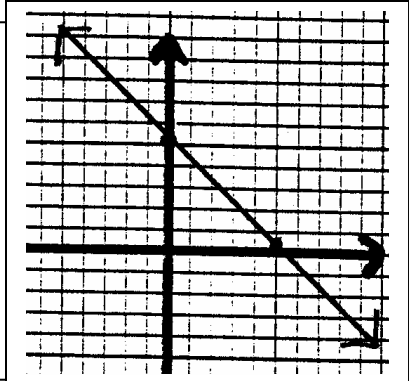
Graph $x + y = 5$ and then graph $2(x + y) = 2(5)$

Rewrite each in slope-intercept form:

$$x + y = 5 \rightarrow y = -x + 5$$

$$2(x + y) = 2(5) \rightarrow 2x + 2y = 10 \rightarrow 2y = -2x + 10 \rightarrow y = -x + 5$$

Both equations have the same slope and y-intercept. They are the same equations.



8. Solve the system of equations by elimination: $2x + 3y = 2$
 $3x - 4y = -14$

Adding the two equations does not eliminate either of the variables. However, if the first equation is multiplied by 4 and the second equation is multiplied by 3, the variable y can be eliminated by addition.

$$4(2x + 3y) = 4(2) \rightarrow 8x + 12y = 8$$

$$3(3x - 4y) = 3(-14) \rightarrow 9x - 12y = -42$$

Add to eliminate y

$$8x + 12y = 8$$

$$\underline{9x - 12y = -42}$$

$$17x = -34$$

$$x = -2$$

Find y by substituting -2 for x .

$$2(-2) + 3y = 2$$

$$3y = 6$$

$$y = 2$$

The solution is $(-2, 2)$

Discuss with students that there are other combinations of multipliers that can be used. For example, the first equation can be multiplied by 9 and the second equation by -6 . You may want students to suggest other multipliers.

9. **Example** – Solve the system of equations by elimination.

$$x + 2y = 11$$

$$x - 4y = 2$$

Multiply $x - 4y = 2$ by negative 1 and add it to first equation.

$$-1(x - 4y) = -1(2) \rightarrow -x + 4y = -2$$

$$x + 2y = 11$$

$$\underline{-x + 4y = -2}$$

$$6y = 9$$

$$y = \frac{3}{2}$$

Substitute $\frac{3}{2}$ for y .

$$x + 2\left(\frac{3}{2}\right) = 11$$

$$x + 3 = 11$$

$$x = 8$$

The solution is $(8, \frac{3}{2})$

10. **Example** – Solve the system of equations by elimination. $4x + 3y = -1$
 $7x + 2y = 1.5$

Multiply top equation by 2 and the bottom equation by -3 and add the two equations.

$$\begin{aligned} 2(4x + 3y) &= 2(-1) \rightarrow 8x + 6y = -2 \\ -3(7x + 2y) &= -3(1.5) \rightarrow \underline{-21x - 6y = -4.5} \\ \hline -13x &= -6.5 \end{aligned}$$

$x = 0.5$

Substitute 0.5 for x.

$$\begin{aligned} 4(0.5) + 3y &= -1 \rightarrow 2 + 3y = -1 \\ 3y &= -3 \\ y &= -1 \end{aligned}$$

Solution is (0.5, -1)

11. **Example** – Solve the system of equations by elimination

Line up the variables.

$$\begin{aligned} \frac{1}{2}x &= \frac{1}{3}y - 1 & \frac{1}{2}x &= \frac{1}{3}y - 1 \\ \frac{1}{5}y &= \frac{3}{4}x - 3 & -\frac{3}{4}x &= -\frac{1}{5}y - 3 \end{aligned}$$

$$\begin{aligned} \frac{1}{2}x &= \frac{1}{3}y - 1 \\ \frac{1}{5}y &= \frac{3}{4}x - 3 \end{aligned}$$

Multiply top equation by 6 and second equation by 20.

$$\begin{aligned} 6\left(\frac{1}{2}x = \frac{1}{3}y - 1\right) &\rightarrow 3x = 2y - 6 \\ 20\left(-\frac{3}{4}x = -\frac{1}{5}y - 3\right) &\rightarrow \underline{-15x = -4y - 60} \end{aligned}$$

Multiply top equation by 2 and add equations

$$\begin{aligned} 6x &= 4y - 12 \\ \underline{-15x} &= \underline{-4y - 60} \\ \hline -9x &= -72 \\ x &= 8 \end{aligned}$$

Substitute 8 for x.

$$\frac{1}{2}(8) = \frac{1}{3}y - 1 \rightarrow 4 = \frac{1}{3}y - 1 \rightarrow y = 15$$

Solution is (8, 15)

Name: _____

Date: _____

Class: _____

Solving Systems of Equations Algebraically Worksheet

Solve each system of equations by the substitution method.

1. $y = 3x$
 $x + 2y = -21$

3. $x - 2y = 5$
 $3x - 5y = 8$

2. $2x + 2y = 4$
 $x - 2y = 0$

4. $3x + 4y = -7$
 $2x + y = -3$

Solve each system of equations by the elimination method.

5. $4x + y = 9$
 $3x - 2y = 4$

7. $4x - 3y = -4$
 $3x - 2y = -4$

6. $2x + y = 0$
 $5x + 3y = 1$

Solve each system of equations. (Use either method).

8. $3x + 2y = 40$
 $x - 7y = -2$

10. $\frac{2x + y}{3} = 15$

9. $x + y = 6$
 $x - y = 4.5$

$\frac{3x - y}{5} = 1$

Write a system of equations and then solve each problem.

11. The sum of two numbers is 42. Their difference is 12. What are the two numbers?
12. The sum of Kate's age and her mother's age is 52. Kate's mother is 20 years older than Kate. How old is each?
13. The perimeter of a rectangle is 86 cm. Twice the width exceeds the length by 2 cm. Find the dimensions of the rectangle.
14. The hypotenuse of a right triangle measures 75 m. The length of one leg is four times one-third the other leg. What are the lengths of the legs? (Pythagorean Theorem: $a^2 + b^2 = c^2$).

Solving Systems of Equations Algebraically Worksheet Key

Solve each system of equations by the substitution method.

1. $y = 3x$
 $x + 2y = -21$

Substitute $3x$ for y in second equation $x + 2(3x) = -21 \rightarrow 7x = -21 \rightarrow x = -3$ substitute -3 for $x \rightarrow y = 3(-3) = -9$

The solution is $(-3, -9)$

2. $2x + 2y = 4$
 $x - 2y = 0$

Solve both equations for $x \rightarrow$

$$\begin{aligned} x &= -y + 2 \\ x &= 2y \end{aligned}$$

$$-y + 2 = 2y \rightarrow 2 = 3y \rightarrow y = \frac{2}{3}$$

$$\text{Substitute } \frac{2}{3} \text{ for } y \rightarrow x - 2\left(\frac{2}{3}\right) = 0 \rightarrow x = \frac{4}{3}$$

The solution is

$$\left(\frac{4}{3}, \frac{2}{3}\right)$$

3. $x - 2y = 5$
 $3x - 5y = 8$

Solve first equation for $x \rightarrow x = 2y + 5$ substitute $2y + 5$ for x in second equation $3(2y + 5) - 5y = 8 \rightarrow y + 15 = 8 \rightarrow y = -7$

Substitute -7 for y in first equation $\rightarrow x - 2(-7) = 5 \rightarrow x + 14 = 5 \rightarrow x = -9$.

The solution is $(-9, -7)$

4. $3x + 4y = -7$
 $2x + y = -3$

Solve the second equation for $y \rightarrow y = -2x - 3$ substitute $-2x - 3$ for y in the first equation $\rightarrow 3x + 4(-2x - 3) = -7 \rightarrow -5x - 12 = -7 \rightarrow 5x = -5 \rightarrow x = -1 \rightarrow$ substitute -1 for x in the second equation $\rightarrow 2(-1) + y = -3 \rightarrow y = -1$

The solution is $(-1, -1)$

Solve each system of equations by the elimination method.

5. $4x + y = 9$
 $3x - 2y = 4$

Multiply top equation by 2 and then add the two equations together.

$$\begin{aligned} 8x + 2y &= 18 \\ \underline{3x - 2y} &= \underline{4} \\ 11x &= 22 \\ x &= 2 \end{aligned}$$

Substitute 2 for x in the first equation.

$$4(2) + y = 9 \rightarrow y = 1$$

The solution is $(2, 1)$

6. $2x + y = 0$
 $5x + 3y = 1$

Multiply first equation by -3 and then add the two equations together.

$$\begin{aligned} -6x - 3y &= 0 \\ \underline{5x + 3y} &= \underline{1} \\ -x &= 1 \\ x &= -1 \end{aligned}$$

Substitute -1 for x in the first equation.

$$2(-1) + y = 0 \rightarrow y = 2$$

The solution is $(-1, 2)$

7. $4x - 3y = -4$
 $3x - 2y = -4$

Multiply first equation by 2 and the second equation by -3 and then add the two equations together.

$$\begin{array}{r} 8x - 6y = -8 \\ \underline{-9x + 6y = 12} \\ -x = 4 \\ x = -4 \end{array}$$

Substitute -4 for x in the second equation.
 $3(-4) - 2y = -4 \rightarrow -2y = 8 \rightarrow y = -4$

The solution is $(-4, -4)$

Solve each system of equations. (Use either method).

8. $3x + 2y = 40$
 $x - 7y = -2$

Methods will vary!

Multiply second equation by -3 and add the two equations.

$$\begin{array}{r} -3x + 21y = 6 \\ \underline{3x + 2y = 40} \\ 23y = 46 \\ y = 2 \end{array}$$

Substitute 2 for y in the second equation.
 $x - 7(2) = -2 \rightarrow x = 12$

The solution is $(12, 2)$

9. $x + y = 6$
 $x - y = 4.5$

Methods will vary!

$$\begin{array}{r} x + y = 6 \\ \underline{x - y = 4.5} \\ 2x = 10.5 \\ x = 5.25 \end{array}$$

Substitute 5.25 for x in the first equation.
 $5.25 + y = 6$
 $y = 0.75$

The solution is $(5.25, 0.75)$

10. $\frac{2x + y}{3} = 15$
 $\frac{3x - y}{5} = 1$

Methods will vary!

Multiply first equation by 3 and second by 5. Then add the two equations together.

$$\begin{array}{r} 2x + y = 45 \\ \underline{3x - y = 5} \\ 5x = 50 \\ x = 10 \end{array}$$

Substitute 10 for x in the first equation.
 $2(10) + y = 45$
 $y = 25$

The solution is $(10, 25)$

Write a system of equations and then solve each problem.

11. The sum of two numbers is 42. Their difference is 12. What are the two numbers?

Let x and y be the two numbers.					
$x + y = 42$ $x - y = 12$	<table border="1"> <tr> <td>Add the two equations.</td> <td>Substitute 27 for x in the first equation.</td> </tr> <tr> <td> $x + y = 42$ $\underline{x - y = 12}$ $2x = 54$ $x = 27$ </td> <td> $27 + y = 42$ $y = 15$ </td> </tr> </table>	Add the two equations.	Substitute 27 for x in the first equation.	$x + y = 42$ $\underline{x - y = 12}$ $2x = 54$ $x = 27$	$27 + y = 42$ $y = 15$
Add the two equations.	Substitute 27 for x in the first equation.				
$x + y = 42$ $\underline{x - y = 12}$ $2x = 54$ $x = 27$	$27 + y = 42$ $y = 15$				
<table border="1"> <tr> <td>The two numbers are 15 and 27.</td> </tr> </table>		The two numbers are 15 and 27.			
The two numbers are 15 and 27.					

12. The sum of Kate's age and her mother's age is 52. Kate's mother is 20 years older than Kate. How old is each?

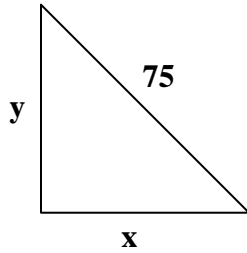
Let x be Kate's age and y be mother's age.					
$x + y = 52$ $y = x + 20$	<table border="1"> <tr> <td>Substitute x + 20 for y in the first equation</td> <td>Substitute 16 for x in the second equation</td> </tr> <tr> <td> $x + (x + 20) = 52 \rightarrow 2x + 20 = 52 \rightarrow x = 16$ </td> <td> $y = 16 + 20 \rightarrow y = 36$ </td> </tr> </table>	Substitute x + 20 for y in the first equation	Substitute 16 for x in the second equation	$x + (x + 20) = 52 \rightarrow 2x + 20 = 52 \rightarrow x = 16$	$y = 16 + 20 \rightarrow y = 36$
Substitute x + 20 for y in the first equation	Substitute 16 for x in the second equation				
$x + (x + 20) = 52 \rightarrow 2x + 20 = 52 \rightarrow x = 16$	$y = 16 + 20 \rightarrow y = 36$				
<table border="1"> <tr> <td> Kate is 16 years of age Mother is 36 years of age </td> </tr> </table>		Kate is 16 years of age Mother is 36 years of age			
Kate is 16 years of age Mother is 36 years of age					

13. The perimeter of a rectangle is 86 cm. Twice the width exceeds the length by 2 cm. Find the dimensions of the rectangle.

Let x be the length and y the width					
<table border="1"> <tr> <td> $2x + 2y = 86$ $2y = x + 2$ </td> <td style="vertical-align: middle;">y cm</td> </tr> </table>	$2x + 2y = 86$ $2y = x + 2$	y cm	<table border="1"> <tr> <td>Solve first equation for 2y</td> </tr> <tr> <td> $2y = -2x + 86$ substitute $-2x + 86$ for $2y$ in the second equation. $-2x + 86 = x + 2 \rightarrow 3x = 84 \rightarrow x = 28$ </td> </tr> </table>	Solve first equation for 2y	$2y = -2x + 86$ substitute $-2x + 86$ for $2y$ in the second equation. $-2x + 86 = x + 2 \rightarrow 3x = 84 \rightarrow x = 28$
$2x + 2y = 86$ $2y = x + 2$	y cm				
Solve first equation for 2y					
$2y = -2x + 86$ substitute $-2x + 86$ for $2y$ in the second equation. $-2x + 86 = x + 2 \rightarrow 3x = 84 \rightarrow x = 28$					
<table border="1"> <tr> <td>Substitute 28 for x in second equation.</td> </tr> <tr> <td> $2y = 28 + 2 \rightarrow y = 15$ </td> </tr> </table>		Substitute 28 for x in second equation.	$2y = 28 + 2 \rightarrow y = 15$		
Substitute 28 for x in second equation.					
$2y = 28 + 2 \rightarrow y = 15$					
<table border="1"> <tr> <td> The length is 28 cm. The width is 15 cm. </td> </tr> </table>		The length is 28 cm. The width is 15 cm.			
The length is 28 cm. The width is 15 cm.					

14. The hypotenuse of a right triangle measures 75 m. The length of one leg is four times one-third the other leg. What are the lengths of the legs?

Let x and y be the legs of a right triangle.



$$x = 4\left(\frac{1}{3}y\right)$$
$$x^2 + y^2 = 75^2$$

Substitute $4\left(\frac{1}{3}y\right)$ for x in the second equation.

$$\left(\frac{4}{3}y\right)^2 + y^2 = 75^2 \Rightarrow \frac{16}{9}y^2 + y^2 = 75^2 \Rightarrow \frac{25}{9}y^2 = 75^2 \Rightarrow$$

$$\sqrt{\frac{25}{9}y^2} = \sqrt{75^2} \Rightarrow \frac{5}{3}y = 75 \Rightarrow y = 45$$

Substitute 45 for y in the first equation.

$$x = 4\left(\frac{1}{3}\right)(45) \Rightarrow x = 60$$

The legs are 45 m and 60 m

Student Name: _____

Date: _____

Solving Systems of Equations Algebraically Checklist

1. On questions 1 thru 4, did the student solve the system of equations correctly by the substitution method?
 - a. Yes (20 points)
 - b. 3 out of 4 (15 points)
 - c. 2 out of 4 (10 points)
 - d. 1 out of 4 (5 points)

2. On questions 5 thru 7, did the student solve the system of equations correctly by the elimination method?
 - a. Yes (50 points)
 - b. 2 out of 3 (10 points)
 - c. 1 out of 3 (5 points)

3. On questions 8 thru 10, did the student solve the system of equations correctly using either the substitution or elimination method?
 - a. Yes (15 points)
 - b. 2 out of 3 (10 points)
 - c. 1 out of 3 (5 points)

4. On questions 11 thru 14, did the student set up a correct system of equations to solve the problem?
 - a. Yes (20 points)
 - b. 3 out of 4 (15 points)
 - c. 2 out of 4 (10 points)
 - d. 1 out of 4 (5 points)

5. On questions 11 thru 14, did the student solve the system of equations correctly?
 - a. Yes (20 points)
 - b. 3 out of 4 (15 points)
 - c. 2 out of 4 (10 points)
 - d. 1 out of 4 (5 points)

Total Number of Points _____

NOTE: The sole purpose of this checklist is to aide the teacher in identifying students that need remediation. It is suggested that teacher's devise their own point range for determining grades. In addition, some students need remediation in specific areas. The following checklist provides a means for the teacher to assess which areas need addressing.

1. Does the student need remediation in content (solving system of equations by substitution) for questions 1 thru 4? Yes _____ No _____
2. Does the student need remediation in content (solving system of equations by elimination) for questions 5 thru 7? Yes _____ No _____
3. Does the student need remediation in content (determining which method to use when solving a system of equations) for questions 8 thru 10? Yes _____ No _____
4. Does the student need remediation in content (writing a system of equations from word problems) for questions 11 thru 14? Yes _____ No _____
5. Does the student need remediation in content (solving a system of equations) for questions 11 thru 14? Yes _____ No _____

- A 117 points and above
- B 107 points and above
- C 100 points and above
- D 88 points and above
- F 95 points and below



Sample point scale.