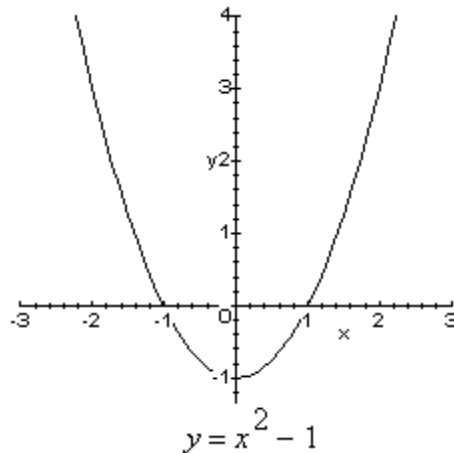


Complexity Examples

1. Why complex numbers?

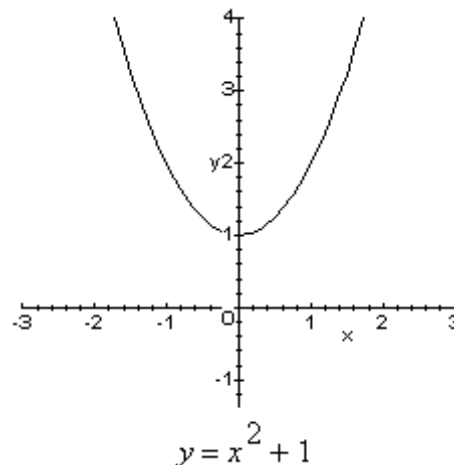
Equation 1: $x^2 - 1 = 0$.

Equation 1 has two solutions, $x = -1$ and $x = 1$. We know that solving an equation in x is equivalent to finding the x -intercepts of a graph; and, the graph of $y = x^2 - 1$ crosses the x -axis at $(-1,0)$ and $(1,0)$.



Equation 2: $x^2 + 1 = 0$

Equation 2 has no solutions, and we can see this by looking at the graph of $y = x^2 + 1$.



Since the graph has no x -intercepts, the equation has no solutions. When we define complex numbers, equation 2 will have two solutions.

2. Numbers such as $5i$ and $27 - 2i$ are imaginary numbers since they contain the imaginary unit i . The real numbers together with the imaginary numbers form the set of **complex numbers**.

Definition of Complex Number	A complex number is any number that can be written in the form of $a + bi$ where a and b are real numbers and i is the imaginary unit, a is called the real part, and bi is called the imaginary part.
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3. Any real number is also a complex number. For example, $\sqrt{2}$ can be written as $\sqrt{2} + 0i$. Its imaginary part is $0i$ or 0 . A complex number is a real number only if its imaginary part is 0 .

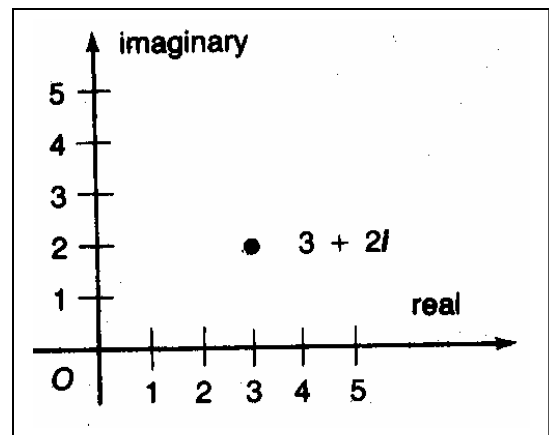
If $b \neq 0$, the complex number $a + bi$ is also called an imaginary number.

4. Any two complex numbers denoted by $a + bi$ and $c + di$, are equal if and only if their real parts are equal and their imaginary parts are equal.

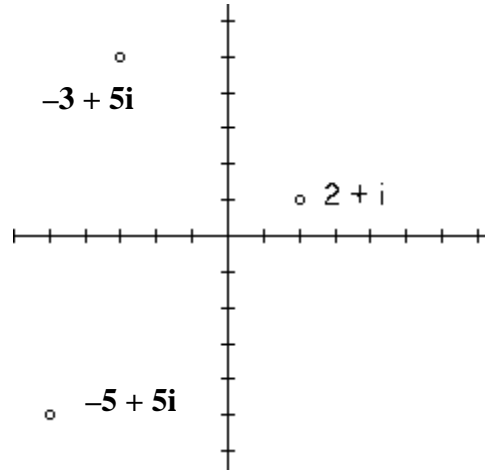
$$\boxed{a + bi = c + di \text{ if and only if } a = c \text{ and } b = d}$$

5. Some students may be confused about the difference between complex and imaginary numbers. The set of complex numbers has two independent subsets – the real and the imaginary numbers. Numbers such as $7 - 3i$ and $8i$ are both complex and imaginary. Real numbers such as $\sqrt{2}$ and 6 are complex but not imaginary. To illustrate this, show a horizontal number line labeled real and a vertical one labeled imaginary

The graph of a complex number $a + bi$ corresponds to the ordered pair (a, b) . The entire plane represents the set of complex numbers. All points not on the real axis are imaginary. All points on the vertical axis, except 0 , are pure imaginary.



6. We can visualize complex numbers by associating them with points in the plane. We do this by letting the *number* $a + bi$ correspond to the *point* (a,b) .



7. **Example** – Find the values for x and y such that $2x + 3yi = 6 + 2i$.

$2x + 3yi = 6 + 2i$ $2x = 6$ and $3y = 2$ $x = 3$ and $y = \frac{2}{3}$	Check $2(3) + 3\left(\frac{2}{3}\right)i = 6 + 2i$ $6 + 2i = 6 + 2i$
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8. To add or subtract complex numbers, combine their real parts and combine their imaginary parts. To multiply complex numbers, use the FOIL method.

9. **Example** – Simplify $(3 + 6i) + (7 - 2i)$.

$$\begin{aligned} & (3 + 6i) + (7 - 2i) \\ & (3 + 7) + (6i - 2i) \\ & 10 + 4i \end{aligned}$$

10. **Example** – Simplify $(6 - 5i) - (3 - 2i)$.

$$\begin{aligned} & (6 - 5i) - (3 - 2i) \\ & (6 - 3) + (-5i - (-2i)) \\ & 3 - 3i \end{aligned}$$

FOIL Method

11. **Example** – Simplify $(6 - 7i)(4 + 3i)$.

$$(6 - 7i)(4 + 3i) \rightarrow 6(4) + 6(3i) + (-7i)(4) + (-7i)(3i) \rightarrow$$

$$24 + 18i - 28i - 21i^2$$

$$(24 + 21) + (18i - 28i)$$

$$45 - 10i$$

$$-21i^2 = 21$$

$$i = \sqrt{-1}$$

$$i^2 = -1$$

12. This chart summarizes addition, subtraction, and multiplication of complex numbers.

For any complex numbers $a + bi$ and $c + di$

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

13. **Example** – Simplify $(8 + 7i) + (-12 + 11i)$

$$(8 + 7i) + (-12 + 11i)$$

$$(8 + (-12)) + (7i + 11i)$$

$$-4 + 18i$$

14. **Example** – Simplify $(-7 + 4i) - (3 + 2i)$

$$(-7 + 4i) - (3 + 2i)$$

$$(-7 - 3) + (4i - 2i)$$

$$-10 + 2i$$

15. **Example** – Simplify $(8 + 5i)(2 - 3i)$

$$(8 + 5i)(2 - 3i)$$

$$(8)(2) + (8)(-3i) + (5i)(2) + (5i)(-3i)$$

$$16 - 24i + 10i - 15i^2$$

$$(16 + 15) + (-24i + 10i)$$

$$31 - 14i$$

16. **Example** – Simplify $(8 + 3i)^2$

$$\begin{aligned} & (8 + 3i)(8 + 3i) \\ & (8)(8) + (8)(3i) + (3i)(8) + (3i)(3i) \\ & 64 + 24i + 24i + 9i^2 \\ & (64 - 9) + (24i + 24i) \\ & 55 + 48i \end{aligned}$$

17. **Example** – Simplify $(2 - i)(3 + i)(4 + 6i)$

Multiply first two quantities. Take their product and multiply by the third quantity.

$$(2 - i)(3 + i) \rightarrow (2)(3) + (2)(i) + (-i)(3) + (-i)(i) \rightarrow (6 + 1) + (2i - 3i) \rightarrow 7 - i$$

$$(7 - i)(4 + 6i) \rightarrow (7)(4) + (7)(6i) + (-i)(4) + (-i)(6i) \rightarrow (28 + 6) + (42i - 4i) \rightarrow$$

$$34 + 38i$$

Name: _____

Date: _____

Class: _____

Complexity Worksheet

Simplify

1. $(6 + 3i) + (2 + 8i)$

2. $(4 - i) + (3 + 3i)$

3. $(2 + 4i) - (2 + 2i)$

4. $(7 - 6i) - (5 - 6i)$

5. $(7 + 3i) + (3 - 3i)$

6. $(2 + 4i) + (2 - 4i)$

7. $4(5 + 3i)$

8. $-6(2 - 3i)$

9. $(1 + 3i)(2 + 4i)$

10. $(2 - 3i)(1 - 4i)$

11. $(3 + 2i)(4 - i)$

12. $(6 + i)(6 - i)$

13. $(6 - 2i)^2$

14. $(2 - \sqrt{-3})(2 + \sqrt{-3})$

15. $(2 + i)(3 - 4i)(1 + 2i)$

Find values of x and y for which each sentence is true (x and y represent real numbers).

16. $x - yi = 5 + 6i$

17. $x + yi = 2 - 3i$

18. $x - yi = 7 - 2i$

19. $x - yi = 4 + 5i$

20. $x + 2yi = 3$

21. $2x + yi = 5i$

22. $2x + 5yi = 4 + 15i$

23. $3x + 2yi = 18 + 7i$

24. Write an expression for the additive inverse of the complex number $a + bi$.

25. Show that 0 is the additive identity for complex numbers.

26. Show that 1 is the multiplicative identity for complex numbers.

Complexity Worksheet Key

Simplify

- $(6 + 3i) + (2 + 8i) \rightarrow (6 + 2) + (3i + 8i) \rightarrow 8 + 11i$
- $(4 - i) + (3 + 3i) \rightarrow (4 + 3) + (-i + 3i) \rightarrow 7 + 2i$
- $(2 + 4i) - (2 + 2i) \rightarrow (2 - 2) + (4i - 2i) \rightarrow 2i$
- $(7 - 6i) - (5 - 6i) \rightarrow (7 - 5) + (-6i - (-6i)) \rightarrow 2$
- $(7 + 3i) + (3 - 3i) \rightarrow (7 + 3) + (3i - 3i) \rightarrow 10$
- $(2 + 4i) + (2 - 4i) \rightarrow (2 + 2) + (4i - 4i) \rightarrow 4$
- $4(5 + 3i) \rightarrow 4(5) + 4(3i) \rightarrow 20 + 12i$
- $-6(2 - 3i) \rightarrow -6(2) + -6(-3i) \rightarrow -12 + 18i$
- $(1 + 3i)(2 + 4i) \rightarrow (1)(2) + (1)(4i) + (3i)(2) + (3i)(4i) \rightarrow 2 + 4i + 6i + 12i^2 \rightarrow -10 + 10i$
- $(2 - 3i)(1 - 4i) \rightarrow (2)(1) + (2)(-4i) + (-3i)(1) + (-3i)(-4i) \rightarrow 2 - 8i - 3i + 12i^2 \rightarrow -10 - 11i$
- $(3 + 2i)(4 - i) \rightarrow (3)(4) + (3)(-i) + (2i)(4) + (2i)(-i) \rightarrow 12 - 3i + 8i - 2i^2 \rightarrow 15 + 5i$
- $(6 + i)(6 - i) \rightarrow (6)(6) + (6)(-i) + (i)(6) + (i)(-i) \rightarrow 36 - 6i + 6i - i^2 \rightarrow 37$
- $(6 - 2i)^2 \rightarrow (6 - 2i)(6 - 2i) \rightarrow (6)(6) + (6)(-2i) + (6)(-2i) + (-2i)(-2i) \rightarrow 32 - 24i$
- $(2 - \sqrt{-3})(2 + \sqrt{-3}) \rightarrow (2 - \sqrt{3}i)(2 + \sqrt{3}i) \rightarrow 2(2) + 2(\sqrt{3}i) + (-\sqrt{3}i)(2) + (-\sqrt{3}i)(\sqrt{3}i) \rightarrow 4 + 2\sqrt{3}i - 2\sqrt{3}i - 3i^2 \rightarrow 7$
- $(2 + i)(3 - 4i)(1 + 2i)$

Multiply first two. Take this product times the third quantity.

$$(2 + i)(3 - 4i) \rightarrow 6 - 5i - 4i^2 \rightarrow 10 - 5i$$

$$(10 - 5i)(1 + 2i) \rightarrow 10 + 15i - 10i^2 \rightarrow 20 + 15i$$

Find values of x and y for which each sentence is true (x and y represent real numbers).

16. $x - yi = 5 + 6i$

$$x = 5$$

$$-y = 6 \rightarrow y = -6$$

17. $x + yi = 2 - 3i$

$$x = 2$$

$$y = -3$$

18. $x - yi = 7 - 2i$

$$x = 7$$

$$-y = -2 \rightarrow y = 2$$

19. $x - yi = 4 + 5i$

$$x = 4$$

$$-y = 5 \rightarrow y = -5$$

20. $x + 2yi = 3$

$$x = 3$$

$$2y = 0 \rightarrow y = 0$$

21. $2x + yi = 5i$

$$2x = 0 \rightarrow x = 0$$

$$y = 5$$

22. $2x + 5yi = 4 + 15i$

$$2x = 4 \rightarrow x = 2$$

$$5y = 15 \rightarrow y = 3$$

23. $3x + 2yi = 18 + 7i$

$$3x = 18 \rightarrow x = 6$$

$$2y = 7 \rightarrow y = \frac{7}{2}$$

24. Write an expression for the additive inverse of the complex number $a + bi$.

$$-a - bi$$

25. Show that 0 is the additive identity for complex numbers.

$$a + bi + 0 + 0i = (a + 0) + (bi + 0i)$$

$$a + bi + 0 + 0i = a + (b + 0)i$$

$$a + bi + 0 + 0i = a + bi$$

26. Show that 1 is the multiplicative identity for complex numbers.

$$(a + bi)(1) = (a)(1) + (bi)(1)$$

$$(a + bi)(1) = a + bi$$

Student Name: _____

Date: _____

Complexity Checklist

1. On questions 1 thru 5, did the student simplify the expression correctly?
 - a. Yes (75 points)
 - b. 14 out of 15 (70 points)
 - c. 13 out of 15 (65 points)
 - d. 12 out of 15 (60 points)
 - e. 11 out of 15 (55 points)
 - f. 10 out of 15 (50 points)
 - g. 9 out of 15 (45 points)
 - h. 8 out of 15 (40 points)
 - i. 7 out of 15 (35 points)
 - j. 6 out of 15 (30 points)
 - k. 5 out of 15 (25 points)
 - l. 4 out of 15 (20 points)
 - m. 3 out of 15 (15 points)
 - n. 2 out of 15 (10 points)
 - o. 1 out of 15 (5 points)

2. On questions 16 thru 23, did the student solve for x and y correctly?
 - a. Yes (40 points)
 - b. 7 out of 8 (35 points)
 - c. 6 out of 8 (30 points)
 - d. 5 out of 8 (25 points)
 - e. 4 out of 8 (20 points)
 - f. 3 out of 8 (15 points)
 - g. 2 out of 8 (10 points)
 - h. 1 out of 8 (5 points)

3. On question 24, did the student write an expression for the additive inverse?
 - a. Yes 5 points

4. On question 25, did the student write an expression for the additive identity?
 - a. Yes 5 points

5. On question 26, did the student show that 1 is the multiplicative identity for complex numbers?
 - a. Yes 5 points

Total Number of Points _____

Student Name: _____

Date: _____

NOTE: The sole purpose of this checklist is to aide the teacher in identifying students that need remediation. It is suggested that teacher's devise their own point range for determining grades. In addition, some students need remediation in specific areas. The following checklist provides a means for the teacher to assess which areas need addressing.

1. Does the student need remediation in content (operations involving complex numbers) for questions 1 thru 15? Yes_____ No_____
2. Does the student need remediation in content (solving equations involving complex numbers) for questions 16 thru 23? Yes_____ No_____
3. Does the student need remediation in content (additive inverses) for question 24? Yes_____ No_____
4. Does the student need remediation in content (additive identities) for question 25? Yes_____ No_____
5. Does the student need remediation in content (multiplicative identities) for question 26? Yes_____ No_____

- A 123 points and above
- B 117 points and above
- C 104 points and above
- D 78 points and above
- F 77 points and below



Sample point scale.