

## Transparency Lesson

Definition of a geometric sequence. "A sequence is said to be geometric if each term, after the first, is obtained by multiplying the preceding term by a common value."

Definition to find the general term of a geometric sequence. The  $n$ th term of a geometric sequence is  $a_n = a_1 r^{n-1}$  where  $a_1$  is the first term and  $r$  is the common ratio. In transparency, explain the derivation of the geometric sequence.

In order to find the  $n$ th term, denoted as  $a_n$  there must be a common ratio, denoted by  $r$ . The first term is written as  $a_1$ .

$$a_1$$

$$a_2 = a_1 r$$

$$a_3 = a_2 r = a_1 r^2$$

$$a_4 = a_3 r = a_1 r^3$$

Notice that the exponent  $r$  is 1 less than the number of the term. Thus, from that observation, the  $n$ th term of the geometric sequence is found and written as:  $a_n = a_1 r^{n-1}$

### Example:

1. Calculate the following 7<sup>th</sup> term of the following geometric sequence. What is  $r$ ?

2, 4, 8...

1, 1/3, 1/9...

5, -5, 5, -5...

6, 9, 27/2...

Solution #1:

A).  $r = 4/2 = 2$

$$a_7 = 2r^{7-1}$$

$$a_7 = 128$$

B).  $r = (1/3)/1 = 1/3$

$$a_7 = (1/3)^{7-1}$$

$$a_7 = 1/729$$

C).  $r = -5/5 = -1$

$$a_7 = 5(-1)^{7-1}$$

$$a_7 = 5$$

D).  $r = 9/6 = 3/2$

$$a_7 = 6(3/2)^{7-1}$$

$$a_7 = 2187/32$$

2. Write the  $n$ th term of the geometric sequence  $a_n = (1/2)^{2n}$  in the form  $a_1 r^{n-1}$  and find the value of  $a_1$  and  $r$ .

Solution #2:

$$a_n = (1/2)^{2n} = (1/4)^n = (1/4)(1/4)^{n-1}$$

$$a_1 = 1/4 \text{ and } r = 1/4$$

3. A geometric sequence consisting of positive numbers has  $a_1 = 18$  and  $a_5 = 32/9$ . Find  $r$ .

Solution #3:

$$n = 5$$

$$a_n = a_1 r^{n-1}$$

$$32/9 = 18 r^4$$

$$r^4 = 16/81$$

$$r = \pm 2/3$$

Since terms are positive,  $r = 2/3$

4. How much will a student pay on the 30<sup>th</sup> day if she or he pays you \$0.01 on the first day, \$0.02 on the second day, and \$0.04 on the third day and so on and so forth?

Solution #4:

$$a_{30} = 0.01(2)^{29}$$

$$a_{30} = 5,368,709.12$$

Definition of the sum of a geometric sequence.  $S_n = a_1 (1 - r^n) / (1 - r)$

Derivation of the formula:

$$S_n = a_1 + a_1 r + a_1 r^2 + \dots + a_1 r^{n-2} + a_1 r^{n-1}$$

$$r S_n = a_1 r + a_1 r^2 + a_1 r^3 + \dots + a_1 r^{n-1} + a_1 r^n$$

$$S_n - r S_n = a_1 - a_1 r^n$$

$$(1-r) S_n = a_1(1 - r^n)$$

$$S_n = a_1 (1 - r^n) / (1 - r)$$

**Examples:**

1. Find sum for the first 7 terms of the following geometric sequence.

$$2, 4, 8, \dots$$

$$1, 1/3, 1/9, \dots$$

$$5, -5, 5, -5, \dots$$

$$6, 9, 27/2, \dots$$

Solution #1:

$$A). S_7 = 2 (1 - 2^7) / (1 - 2) = 254$$

$$B). S_7 = (1 - (1/3)^7) / (1 - (1/3)) = 1.499$$

$$C). S_7 = 5 (1 - (-1)^7) / (1 - (-1)) = 5$$

$$D). S_7 = 6 (1 - (3/2)^7) / (1 - (3/2)) = 193.03$$

2. How much total must a student pay if the first day, s/he brings in \$0.01, the second day, \$0.02, and so on for 30 days?

Solution #2:

$$S_{30} = 0.01 (1 - (2)^{30}) / (1 - 2) = \$10,737,418.23$$

## Student Worksheet

### Geometric Sequence

1. Find the eighth term of 4, 8, 16, 32, ...
2. Write the fourth term after the first term and the common ratio are given:
  - A)  $a = 2, r = 3$
  - B)  $a = \frac{1}{2}, r = 4$
  - C)  $a = -27, r = \frac{2}{3}$
3. Each year, the value of a car is 70% of its value of the previous year. At the end of the first year, the value of the car was \$6,000. What was its value at the end of 3 years?
4. If you were given a gift of \$1 on your first birthday, and the gift was doubled on each of the following birthday, how much would you receive on your twenty-first birthday?
5. The value of a certain rare coin increases by one-tenth each year. If the coin is worth \$3.00 now, what will be its approximate value in 5 years?

### Sum of Geometric Sequence

1. Find the sum of the first 8 terms of 4, 8, 16, 32, ...
2. Suppose that you save \$128 in January and that each month thereafter you only manage to save half of what you saved the previous month. How much do you save in the tenth month, and what are your total savings after 10 months.
3. In a lottery, the first ticket drawn paid a prize of \$30,000. Each succeeding ticket paid half as much as the preceding one. If six tickets were drawn, what was the total of prize money paid?
4. The half-life of Uranium 230 isotope is 20.8 days, that is, one-half of a given amount of Uranium 230 decomposes every 20.8 days. How much of an initial amount of 1000 grams of the isotope will be left after 208 days?

## Solutions for the Student Worksheet

### Geometric Sequence

1. 512
2. A). 54 B). 32 C). -8
3. 2940
4. \$1,048,576
5. \$4.83

### Sum of Geometric Sequence

1. 1020
2. \$255.75
3. \$59,062.50
4. 0.98 grams